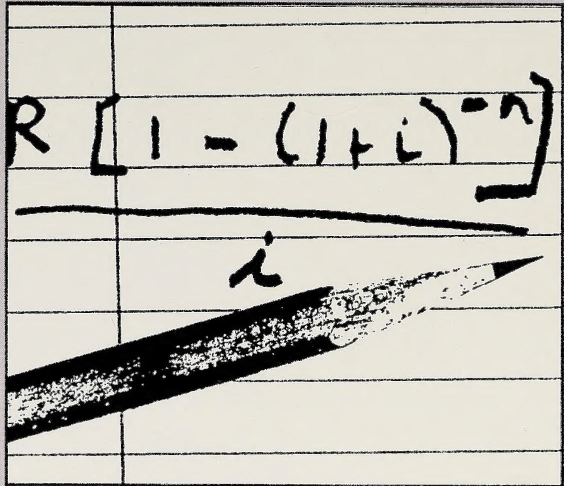
MATHEMATICS 3

Annuities



Unit 7

Distance Learning

Alberta EDUCATION

W e l c o m e

Distance Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

Mathematics 33 Student Module Unit 7 Annuities Alberta Distance Learning Centre ISBN No. 0-7741-0190-3

General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

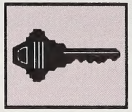
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

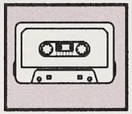
Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.

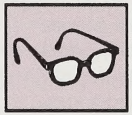


Key Idea
- flagging important ideas

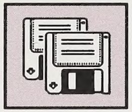


Audiotape
- learning by listening to an audiotape

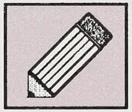
What You Already Know
- reviewing what you already know



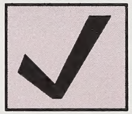
Another View
- exploring different perspectives



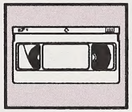
Computer Software
- learning by using computer software



Review
- studying previous concepts

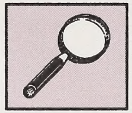


Solutions
- correcting the activities

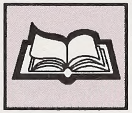


Videotape
- learning by viewing a videotape

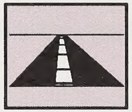
Introduction
- introducing the unit



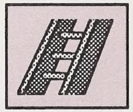
Extra Help
- providing additional study



Print Pathway
- choosing a print alternative



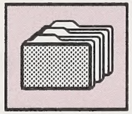
What Lies Ahead
- previewing the unit



Extensions
- going on with the topic

Calculator
- using your calculator

Exploring the Topic
- actively learning new concepts



What You Have Learned
- summarizing what you have learned

Mathematics 33

Course Overview

Mathematics 33 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Polynomials and Rational Expressions	10%
Unit 3 Functions and Relations	16%
Unit 4 Quadratic Functions and Equations	20%
Unit 5 Trigonometry	16%
Unit 6 Statistics	16%
Unit 7 Annuities	6%
Unit 8 Mortgages and Loans	6%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Annuities

This unit covers topics dealing with annuities. Each topic contains explanations, examples, and activities to assist you in understanding annuities. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in **Appendix A**. In several cases there is more than one way to do the question.

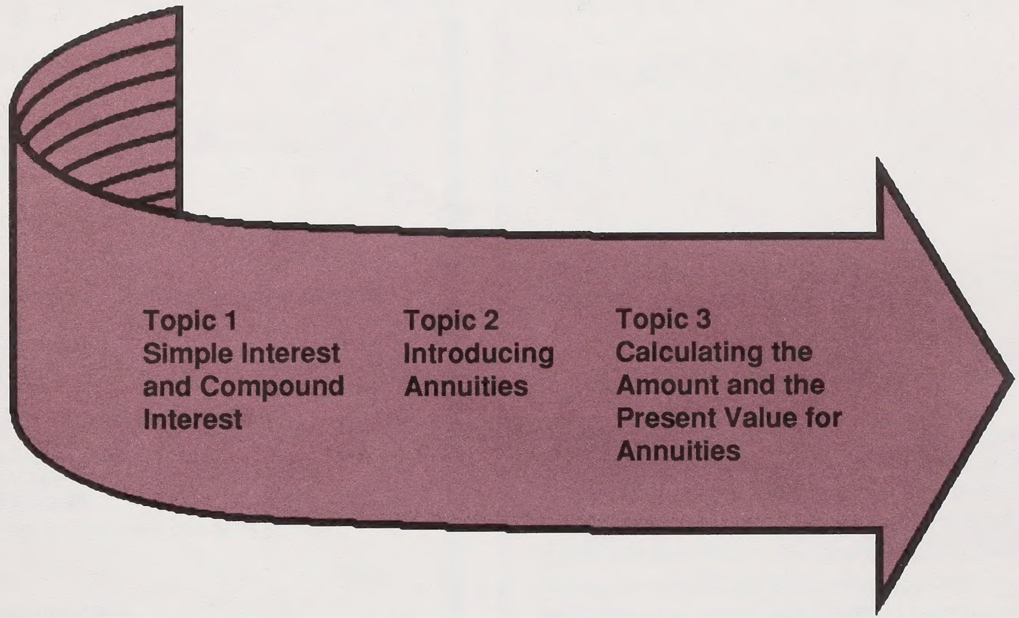
Unit 7 Annuities

Contents at a Glance

Annuities

If people have money that they do not wish or need to spend, they would be wise to rent it out. If this choice is made, the people are making an investment. Investments come in various forms or plans. Some common investments are as follows: savings accounts, bonds, stocks, term deposits, antiques, education, livestock, annuities, and precious metals. In this unit you will study the various aspects of investments known as annuities.

Unit 7 Annuities



What You Already Know

Recall the following.

- Percents can be changed to decimal numbers.

$$5\% = \frac{5}{100} = 0.05$$

$$7\frac{1}{2}\% = \frac{7\frac{1}{2}}{100} = \frac{7.5}{100} = 0.075$$

$$0.025\% = \frac{0.025}{100} = 0.000\,25$$

$$\frac{3}{4}\% = \frac{\frac{3}{4}}{100} = \frac{0.75}{100} = 0.0075$$

$$6\frac{2}{9}\% = \frac{6\frac{2}{9}}{100} = \frac{6.\dot{2}}{100} = 0.06\dot{2}$$

- Powers can be calculated using a calculator.

$(2.23)^3$

Enter	Display
C	0
2.23	2.23
x^y	2.23
3	3
=	11.089567

$(1.004)^{32}$

Enter	Display
C	0
1.004	1.004
x^y	1.004
32	32
=	1.136262856

To change any percent to a decimal, move the decimal point two places to the left and drop the percent sign.

$(2.03)^{-4}$

Enter	Display
C	0
2.03	2.03
x^y	2.03
4	4
+/–	–4
=	0.058886514

$(3.3042)^{-5}$

Enter	Display
C	0
3.3042	3.3042
x^y	3.3042
5	5
+/–	–5
=	0.002539033118

- Numbers can be rounded or approximated to the required number of decimal places or to the required place holder.

15.056 42

to the nearest ten is 20	
to the nearest one is 15	
to the nearest tenth is 15.1	(1 decimal place)
to the nearest hundredth is 15.06	(2 decimal places)
to the nearest thousandth is 15.056	(3 decimal places)
to the nearest ten thousandth is 15.0564	(4 decimal places)

- The symbol ≐ means **is approximately equal to.**
- The +/– key on a calculator changes the sign of the previous entry.

Review

Try the following review questions.

1. Change each of the following to decimal number form.

   a. 5% b. 13%

   c. $10\frac{3}{4}\%$ d. $\frac{2}{3}\%$

   e. $\frac{7}{20}\%$ f. 0.3%

   g. 0.415%

2. Show how each of the following is entered on your calculator.

   a. -0.7
   b. -4.33
   c. -306.44

3. Evaluate each of the following to four decimal places. Use the $\doteq$ symbol where appropriate. Show how a calculator would be used.

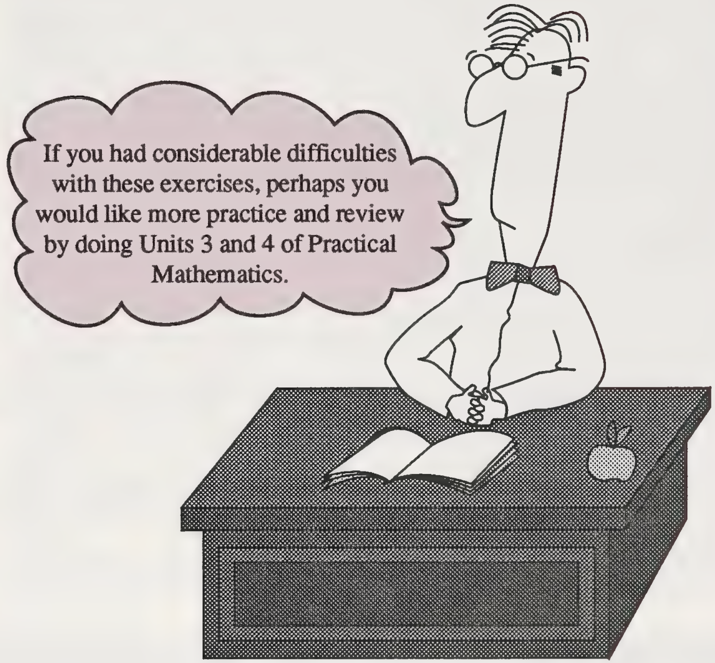
   a. $(1.23)^6$ b. $1500(1.03)^{22}$

   c. $(1.44)^{-4}$ d. $1250(1.36)^{-5}$

4. Round each of the following to the nearest whole number, tenth, hundredth, thousandth, and ten thousandth.

   a. 406.735 28 b. 3342.063 95

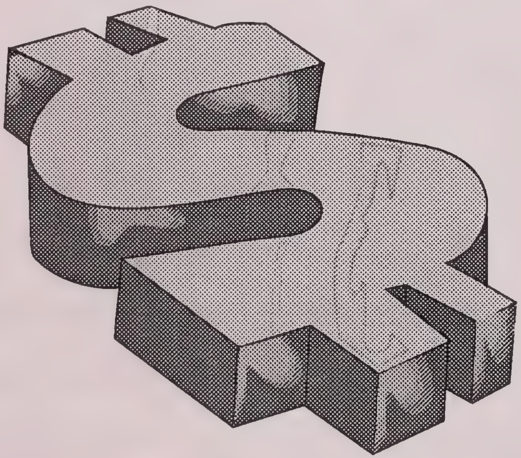
Now go to the **Review** solutions in **Appendix A**.



Topic 1 Simple Interest and Compound Interest

Introduction

When dealing with loans or investments, the understanding of interest and its implications is of great advantage. Interest rates are not the same at all financial institutions. When applying for a loan or when making an investment, shopping around for the best deal is recommended. Interest rates will also vary for different types of loans and investments.



What Lies Ahead

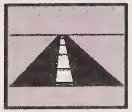
Throughout the topic you will learn to

1. compare the growth of an investment over time using simple and compound interest

Now that you know what to expect, turn the page to begin your study of simple interest and compound interest.

Exploring Topic 1

Activity 1



Compare the growth of an investment over time using simple and compound interest.

Simple Interest

Whenever you borrow money, an extra amount must be paid back for the privilege of using money which is not your own. The extra amount that is added to the original loan is called **interest**. The actual amount of interest depends on the interest rate and the amount of time needed to repay the loan.

On the other hand, if money is put into a savings account or some other type of investment, the bank, credit union, or other financial institution will use your money for its own investments. For this privilege, the financial institution pays you interest. It must be noted that interest rates for loans and deposits are not the same. The interest on deposits is added to the previous amount at regular times during the year.

These times may be monthly, quarterly, or semiannually depending on the type of investment. It can be concluded that the longer an investment stays in effect, the greater the amount of accumulated interest. Study the following example to see how simple interest is calculated and how an amount grows over an extended period of time. As time goes on, interest is paid on interest; thus, the original amount increases.

Example 1

What is the interest on a \$3500 deposit at 9%/a for one year?

Solution:

$$\begin{aligned} I &= Prt \\ &= \$3500 \times 0.09 \times 1 \\ &= \$315 \end{aligned}$$

The simple interest at the end of one year would be \$315.

The amount at the end of one year would be as follows:

$$\begin{aligned} A &= P + I \\ &= \$3500 + \$315 \\ &= \$3815 \end{aligned}$$

9%/a means 9% per annum or 9% per year.

In $I = Prt$,
I = interest
P = principal
r = rate
t = time

In $A = P + I$,
A = amount
P = principal
I = interest

Example 2

What would be the amount at the end of three years on a deposit of $4000 at 9.5%/a? The interest will be added to the amount after the three years have expired.

Solution:

$$
\begin{aligned}
I &= Prt \\
&= \$4000 \times 0.095 \times 3 \\
&= \$1140
\end{aligned}
$$

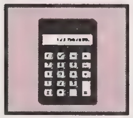
The simple interest at the end of three years would be $1140. The amount at the end of three years would be as follows:

$$
\begin{aligned}
A &= P + I \\
&= \$4000 + \$1140 \\
&= \$5140
\end{aligned}
$$

Example 3

What is the amount at the end of two years on a deposit of $5000 at 11%/a if the interest is added to the previous amount every six months.

Solution:



1. $I = Prt$

$$
\begin{aligned}
&= \$5000 \times 0.11 \times 0.5 \\
&= \$275
\end{aligned}
$$

$$
\begin{aligned}
A &= \$5000 + \$275 \\
&= \$5275
\end{aligned}
$$

2. $I = Prt$

$$
\begin{aligned}
&= \$5275 \times 0.11 \times 0.5 \\
&= \$290.13
\end{aligned}
$$

$$
\begin{aligned}
A &= \$5275.00 + \$290.13 \\
&= \$5565.13
\end{aligned}
$$

3. $I = Prt$

$$
\begin{aligned}
&= \$5565.13 \times 0.11 \times 0.5 \\
&= \$306.08
\end{aligned}
$$

$$
\begin{aligned}
A &= \$5565.13 + \$306.08 \\
&= \$5871.21
\end{aligned}
$$

4. $I = Prt$

$$
\begin{aligned}
&= \$5871.21 \times 0.11 \times 0.5 \\
&= \$322.92
\end{aligned}
$$

$$
\begin{aligned}
A &= \$5871.21 + \$322.92 \\
&= \$6194.13
\end{aligned}
$$

The amount of accumulated interest over the two-year time period is $\$6194.13 - \$5000.00 = \$1194.13$.

In Example 3 the interest is added every six months so the number of periods is $2 \times 2 = 4$.

Your answers may differ slightly from those shown in this unit. Answers are dependent on the type of calculator that is used and at what stage the rounding is done. The best method is to use all digits in the calculator and to round only in the final step to arrive at the final answer.

Using the information from Example 3, what would the amount and total interest be if the interest was not added to the previous amount at the end of each six-month period?

$$\begin{aligned} A &= P(1+ni) \\ &= \$5000(1+4\times 0.055) \\ &= \$5000(1+0.22) \\ &= \$5000(1.22) \\ &= \$6100 \end{aligned}$$

or

$$\begin{aligned} I &= Prt \\ &= \$5000\times 0.11\times 2 \\ &= \$1100 \end{aligned}$$

$$\begin{aligned} A &= P+I \\ &= \$5000+\$1100 \\ &= \$6100 \end{aligned}$$

The amount is \$6100 and the total interest is $\$6100-\$5000=\$1100$.

You can see that it is better when the interest is added to the previous amount every six months and the interest calculation for the next period includes the interest for the previous period. When calculated this way, the total interest is greater compared to adding the interest just once at the end of the entire period. The difference is $\$1194.13-\$1100.00=\$94.13$.

Now try some similar problems on your own.

Complete the odd- or the even-numbered questions from the following list.

1. Find the simple interest and the amount for a deposit of \$6500 at $9\frac{1}{4}\%$ for one year.
2. Find the simple interest and the amount for a deposit of \$10 500 at $8\frac{3}{4}\%$ for one year.
3. Find the simple interest and the amount for a deposit of \$4550 at $12\frac{1}{2}\%$ for four years.
4. Find the simple interest and the amount for a deposit of \$20 150 at $11\frac{1}{5}\%$ for three years.
5. What would the amount be at the end of one year on a deposit of \$1500 at $13\frac{1}{2}\%$ if the interest were to be added to the previous amount every three months? Round the amount of interest to the nearest cent where necessary.

In $A=P(1+ni)$,

A = amount
P = principal
n = number of periods
i = interest per period

In the latter part of Example 3, the 11%/a must be divided by 2 since the simple interest is calculated every six months or twice a year.

$$\begin{aligned} 11\% &= 0.11\div 2 \\ &= 0.055 \end{aligned}$$

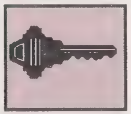
The use of the formula $A=P(1+ni)$ will be dealt with in more detail in the **Extensions** section of this topic.

6. What would the amount be at the end of $1\frac{1}{2}$ years on a deposit of \$1500 at $12\frac{4}{5}\%$ if the interest were to be added to the previous amount every six months? Round the amount of interest to the nearest cent where necessary.

For solutions to **Activity 1**, turn to **Appendix A, Topic 1.**

Compound Interest

In Example 3 the interest earned was added to the previous amount every six months for two years. When the next amount of interest was calculated, the interest from the previous period was included. When this happens, the interest is **compounded**. In this case it was compounded semiannually. The compounding period may vary. It could be 1 year, $\frac{1}{2}$ year, $\frac{1}{3}$ year, $\frac{1}{4}$ year, weekly, or daily. To calculate compound interest, you can repeat the simple interest process as many times as necessary or you can use a special formula developed especially for calculating interest which is compounded over a considerable number of periods. Use the following formula. The use of your calculator is recommended.



$A = P(1+i)^n$, where

$A =$ amount,

$P =$ principal,

$i =$ interest rate per compounding period, and

$n =$ the number of compounding periods

When using the **compound interest formula**, special attention must be centred on the i- and n-values. Study the following example.

Example 4

What would the values be for i and n when calculating compound interest in each of the following instances?

- The rate per annum is $10\frac{1}{2}\%$ and the interest is compounded semiannually for five years.

Solution:

The rate per annum is divided in half since the interest is compounded semiannually.

$$\begin{aligned} i &= \frac{1}{2} \times 10\frac{1}{2}\% \\ &= 0.5 \times 0.105 \\ &= 0.0525 \end{aligned}$$

The total time period is multiplied by 2 since there are two half years in one year.

$$\begin{aligned} n &= 5 \times 2 \\ &= 10 \end{aligned}$$

- The rate per annum is $11\frac{1}{4}\%$ and the interest is compounded monthly for three years.

Solution:

The rate per annum is divided by 12 since the interest is compounded every month.

$$\begin{aligned} i &= \frac{1}{12} \times 11\frac{1}{4}\% \\ &= \frac{1}{12} \times 0.1125 \\ &= 0.009\,375 \end{aligned}$$

The total time period is multiplied by 12 since there are twelve months in one year.

$$\begin{aligned} n &= 3 \times 12 \\ &= 36 \end{aligned}$$

This entire situation will become clearer as you work through the following example. In the example the total interest is calculated using the repeating simple interest formula as well as the compound interest formula. Compare the amount of work that is involved in both methods.

Example 5

Find the total interest earned on a deposit of \$1500 at $9\frac{1}{2}\%$/a compounded every six months for three years. If necessary, round the amount of interest to the nearest cent.

Solution:

The simple interest formula must be used six times.

1. $$\begin{aligned} I &= Prt \\ &= \$1500 \times 0.095 \times 0.5 \\ &= \$71.25 \\ A &= \$1500.00 + \$71.25 \\ &= \$1571.25 \end{aligned}$$

2. $$\begin{aligned} I &= Prt \\ &= \$1571.25 \times 0.095 \times 0.5 \\ &= \$74.63 \\ A &= \$1571.25 + \$74.63 \\ &= \$1645.88 \end{aligned}$$

3. $$\begin{aligned} I &= Prt \\ &= \$1645.88 \times 0.095 \times 0.5 \\ &= \$78.18 \\ A &= \$1645.88 + \$78.18 \\ &= \$1724.06 \end{aligned}$$

A similar pattern would be used regardless of the rate per annum and the number of compounding periods.

4. $I = Prt$

$$= \$1724.06 \times 0.095 \times 0.5$$
$$= \$81.89$$
$$A = \$1724.06 + \$81.89$$
$$= \$1805.95$$

5. $I = Prt$

$$= \$1805.95 \times 0.095 \times 0.5$$
$$= \$85.78$$
$$A = \$1805.95 + \$85.78$$
$$= \$1891.73$$

6. $I = Prt$

$$= \$1891.73 \times 0.095 \times 0.5$$
$$= \$89.86$$
$$A = \$1891.73 + \$89.86$$
$$= \$1981.59$$

The total interest earned is $\$1981.59 - \$1500.00 = \$481.59$.

This method involves a lot of work. There is another method that involves less work.

Use the compound interest formula as shown.

$$P = 1500$$
$$i = 9\frac{1}{2}\% \div 2$$
$$= 0.095 \div 2$$
$$= 0.0475$$
$$n = 3 \times 2$$
$$= 6$$

$$A = P(1+i)^n$$
$$= \$1500(1+0.0475)^6$$
$$= \$1500(1.0475)^6$$
$$= \$1500 \times 1.321\,065\,01$$
$$= \$1981.597\,515$$
$$= \$1981.60$$

The total interest earned is $\$1981.60 - \$1500.00 = \$481.60$.

The two interest amounts are very close to being the same. The difference is due to the rounding which was necessary. Rounding is done to the nearest cent when applying the simple interest formula.

Use your calculator to evaluate this equation.

Enter	Display
$\boxed{C}$	0
1.0475	1.0475
$\boxed{x^y}$	1.0475
6	6
$\boxed{\times}$	1.32106501
1500	1500
$\boxed{=}$	1981.597515

Example 6

Find the interest on a loan of \$4500 at 15%/a compounded monthly for five years.

Solution:

Use $A = P(1+i)^n$.

$$i = \frac{1}{12} \times 15\%$$
$$= \frac{1}{12} \times 0.15$$
$$= 0.0125$$

$$n = 5 \times 12$$
$$= 60$$

$$A = P(1+i)^n$$
$$= \$4500(1+0.0125)^{60}$$
$$= \$4500(1.0125)^{60}$$
$$= \$4500 \times (2.107\,181\,347)$$
$$= \$9482.316\,061$$
$$= \$9482.32$$

Enter	Display
C	0
1.0125	1.0125
x^y	1.0125
60	60
×	2.107181347
4500	4500
=	9482.316061

$$I = \$9482.32 - \$4500.00$$
$$= \$4982.32$$

The amount of interest is \$4982.32.

Can you imagine using the simple interest formula if interest were to be compounded every month for 5 years? The simple interest formula would have to be used 60 times.

Example 7

Find the interest on a deposit of $6500 at $14\frac{3}{4}$%/a compounded daily for the months of May and June.

Solution:

Use the formula $A = P(1+i)^n$.

Divide i by 365 to find the rate per day.

$$\begin{aligned} i &= 14\frac{3}{4}\% \div 365 \\ &= 0.1475 \div 365 \\ &= \frac{0.1475}{305} \\ &= 0.000\,404\,109\,589 \end{aligned}$$

$$\begin{aligned} n &= 31 + 30 \\ &= 61 \end{aligned}$$

$$\begin{aligned} A &= P(1+i)^n \\ &= \$6500(1+0.000\,404\,109\,589)^{61} \\ &= \$6500(1.000\,404\,109\,589)^{61} \\ &= \$6500(1.024\,951\,916) \\ &= \$6662.187\,452 \\ &= \$6662.19 \end{aligned}$$

Enter	Display
C	0
1.00040411	1.00040411
x^y	1.00040411
61	61
×	1.024951947
6500	6500
=	6662.187656

$$\begin{aligned} I &= \$6662.19 - \$6500.00 \\ &= \$162.19 \end{aligned}$$

The amount of interest is $162.19.

The answer you get will depend on how you rounded the numbers. This happened in the previous example as can be seen in the two different numbers. The final answer still rounds to $162.19 for both methods.

Now try some questions on your own.

Do at least five of the parts in question 7. Do questions 8 and 9.

7. Calculate the interest for each of the following investments.

   a. \$1550 at $9\frac{1}{2}$%/a compounded semiannually for three years

   b. \$8335 at $11\frac{1}{4}$%/a compounded every three months for five years

   c. \$935.18 at $13\frac{3}{4}$%/a compounded daily for the months of July, August, and September

   d. \$10 300 at $8\frac{1}{2}$%/a compounded every month for four years

   e. \$7432 at $10\frac{4}{5}$%/a compounded every two months for five years

   f. \$125 000 at $5\frac{7}{8}$%/a compounded every four months for $3\frac{2}{3}$ years

   g. \$525.75 at $12\frac{1}{4}$%/a compounded every six months for $17\frac{1}{2}$ years

8. The Jacoles family invested \$5000 at 11% for four years.

   a. What is the accumulated amount of their investment if the interest is compounded semiannually?

   b. What is the accumulated amount of their investment if the interest is compounded every three months?

9. The Schmidt family needs to borrow \$3500 for three years. At Bank A the rate is $15\frac{1}{2}$%/a compounded quarterly. At Bank B the rate is 16% compounded semiannually. What is the difference between the two amounts they would owe at each bank?

For solutions to **Activity 1,** turn to **Appendix A, Topic 1.**

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
} You may decide to do both.

Extra Help

When calculating simple interest, use the following formula:

$I = Prt$, where
- I = interest,
- P = principal,
- r = rate, and
- t = time

Interest is money, in dollars and cents, paid for a loan or the amount earned when an investment is made.

Principal is the original amount of the loan or the investment.

Rate is usually given as an annual rate. If it is not an annual rate, then a change to an annual rate is needed particularly if the time is in years.

Time is the duration for which the loan or the investment is in effect. The time may be in days, weeks, months, or years. The time and the period of time for which the interest rate is stated must be in the same units. In other words, if the interest rate is per year, the time must be in years. If the interest rate is per month, the time must be in months.

Simple interest is added to the original amount at the end of the time period involved.

Example 8

Find the simple interest and the amount which must be paid back at the end of four years if a \$4000 loan is taken out at $10\frac{1}{2}\%/\text{a}$.

Solution:

To find the interest, use $I = Prt$.

$$\begin{aligned} I &= Prt \\ &= \$4000 \times 0.105 \times 4 \\ &= \$1680 \end{aligned}$$

The amount of interest is \$1680.

To find the amount to be paid back, use $A = P + I$.

$$\begin{aligned} A &= P + I \\ &= \$4000 + \$1680 \\ &= \$5680 \end{aligned}$$

The total amount to be paid back is \$5680.

When calculating compound interest, use the following formula:

$A = P(1+i)^n$, where
A = amount,
P = principal,
i = interest rate per period, and
n = number of periods

The value of i depends on the n-value. If the interest is compounded semiannually, the yearly interest rate is divided by 2. If the time is in years, the n-value is multiplied by 2. If interest is compounded quarterly, i is the yearly interest rate divided by 4 and the n-value is the number of years multiplied by 4. A similar pattern is used when the interest is compounded monthly, weekly, or daily. The only time that i and n do not change occurs when the interest is compounded annually and the interest rate is per year.

Example 9

Find the amount of interest on an investment of \$4700 for two years at $8\frac{1}{2}$%/a when the interest is compounded as follows:

- yearly

Solution:

$$\begin{aligned} A &= P(1+i)^n \\ &= \$4700(1+0.085)^2 \\ &= \$4700(1.085)^2 \\ &= \$4700(1.177\,225) \\ &= \$5532.9575 \\ &= \$5532.96 \end{aligned}$$

$$\begin{aligned} I &= \$5532.96 - \$4700.00 \\ &= \$832.96 \end{aligned}$$

Therefore, the interest amounts to \$832.96.

- semiannually

Solution:

$$\begin{aligned} A &= P(1+i)^n \\ &= \$4700(1+0.0425)^4 \\ &= \$4700(1.0425)^4 \\ &= \$4700(1.181\,147\,825) \\ &= \$5551.394\,778 \\ &= \$5551.39 \end{aligned}$$

$$\begin{aligned} I &= \$5551.39 - \$4700.00 \\ &= \$851.39 \end{aligned}$$

Therefore, the interest amounts to \$851.39.

1 year = 52 weeks

Use a calculator to solve the equation.

Enter	Display
$\boxed{\text{C}}$	0
1.0425	1.0425
$\boxed{x^y}$	1.0425
4	4
$\boxed{\times}$	1.181147825
4700	4700
$\boxed{=}$	5551.394778

- quarterly

Solution:

$$A = P(1+i)^n$$
$$= \$4700(1+0.021\,25)^8$$
$$= \$4700(1.021\,25)^8$$
$$= \$4700(1.183\,195\,628)$$
$$= \$5561.019\,453$$
$$= \$5561.02$$

$$I = \$5561.02 - \$4700.00$$
$$= \$861.02$$

Therefore, the interest amounts to $861.02.

1. Find the simple interest for each of the following investments.

   a. $5600 at $9\frac{1}{4}$%/a for 6 years
   b. $10 100 at $12\frac{3}{4}$%/a for $4\frac{1}{4}$ years
   c. $107 432 at $8\frac{1}{2}$%/a for $3\frac{1}{2}$ years

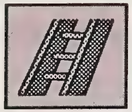
2. Find the compound interest for each of the following deposits.

   a. $5600 at $9\frac{1}{4}$%/a for 6 years compounded yearly
   b. $10 100 at $12\frac{3}{4}$%/a for 4 years compounded quarterly
   c. $107 432 at $8\frac{1}{2}$%/a for $3\frac{1}{2}$ years compounded monthly

For solutions to **Extra Help**, turn to **Appendix A, Topic 1.**

Use a calculator to solve the equation $A = \$4700(1.02125)^8$.

Enter	Display
$\boxed{C}$	0
1.02125	1.02125
$\boxed{x^y}$	1.02125
8	8
$\boxed{\times}$	1.183195628
4700	4700
$\boxed{=}$	5561.019453



Extensions

To calculate simple interest, you can use one of the following formulas.

- $I = Prt$
- $A = P(1+ni)$

Take a closer look at the second formula.

$A = P(1+ni)$, where
- A = amount,
- P = principal,
- n = number of interest periods which may be in years, half years, quarter years, weeks, months, or days, and
- i = interest rate per period
  For example, if time is in years, a rate of 12% means 12% per year, 6% per half year, and 3% per quarter year.

Compare the use of these two formulas to see if the same amount of interest for the same conditions can be found.

Example 10

What is the simple interest on a \$1350 investment at 12%/a for three years if interest is calculated two times a year?

Solution:

Using $I = Prt$,

$$\begin{aligned} &= \$1350 \times 0.12 \times 3 \\ &= \$486 \end{aligned}$$

Using $A = P(1+ni)$,

$$\begin{aligned} &= \$1350(1+6\times 0.06) \\ &= \$1350(1+0.36) \\ &= \$1350\times 1.36 \\ &= \$1836 \end{aligned}$$

$$\begin{aligned} n &= 3\times 2 \\ &= 6 \\ i &= 12\% \div 2 \\ &= 6\% \\ &= 0.06 \end{aligned}$$

The interest would be $\$1836 - \$1350 = \$486$.
In both cases the amount of interest is exactly the same.

Example 11

What would the total interest be for the situation in Example 10 if interest were to be calculated every two months? Use $A = P(1+ni)$.

Solution:

$$\begin{aligned} A &= P(1+ni) \\ &= \$1350(1+18\times 0.02) \\ &= \$1350(1+0.36) \\ &= \$1350\times 1.36 \\ &= \$1836 \end{aligned} \qquad \begin{aligned} n &= 3\times 6 \\ &= 18 \\ i &= 12\% \div 6 \\ &= 2\% \\ &= 0.02 \end{aligned}$$

$$\begin{aligned} I &= \$1836 - \$1350 \\ &= \$486 \end{aligned}$$

The amount of interest is exactly the same as in Example 10.

When working with compound interest problems, it is sometimes necessary to calculate an amount required now to result in a certain total at a later date. Problems like this can be solved by using a formula similar to the compound interest formula.

To calculate the amount in the future when interest is compounded, you use the formula $A = P(1+i)^n$.

To calculate the amount needed now to result in a future total, you use the formula $PV = A(1+i)^{-n}$, where

PV = present value,
A = future amount,
i = interest rate per period, and
n = number of interest periods.

Note: The amount needed now can be called the present value or *PV*.

Example 12

Find the amount of money needed now that will yield a total of \$500 in two years if the interest rate is 8%/a compounded semiannually.

Solution:

$$
\begin{aligned}
PV &= A(1+i)^{-n} \\
&= \$500(1+0.04)^{-4} \\
&= \$500 \times \frac{1}{(1+0.04)^4} \\
&= \$500 \times \frac{1}{(1.04)^4} \\
&= \$500 \times \frac{1}{1.169\,858\,56} \\
&= \frac{\$500}{1.169\,858\,56} \\
&= \$427.402\,095\,6 \\
&= \$427.40
\end{aligned}
$$

The amount needed now to result in \$500 in two years is \$427.40.

For more practice, do the following problems.

1. a. Use $I = Prt$ to find I when $P = \$1050$, $r = 7\frac{1}{2}\%$, and $t =$ four years.

   b. Use $A = P(1+ni)$ to find I when $P = \$1050$, $r = 7\frac{1}{2}\%$, and $t =$ four years. The interest is calculated twice a year.

2. Find the present value for \$1225 at $9\frac{1}{2}\%/a$ compounded semiannually for four years.

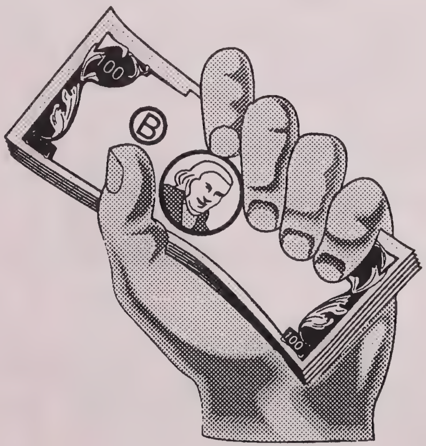
3. Find the present value for \$5350 at $11\frac{3}{4}\%/a$ compounded every month for five years.

For solutions to **Extensions**, turn to **Appendix A, Topic 1.**

Topic 2 Introducing Annuities

Introduction

Financial institutions such as banks, credit unions, trust companies, and insurance companies offer many forms of investments. One investment which is safe and relatively easy to acquire is an annuity. The decision to invest in an annuity takes commitment and discipline since regular contributions on a regular basis for a long time are required. In this topic you will learn more about annuities.



What Lies Ahead

Throughout the topic you will learn to

1. define annuity
2. develop a repeating process to find the amount and the present value of an annuity

Now that you know what to expect, turn the page to begin your study of annuities.

Exploring Topic 2

Activity 1

Define annuity.

An annuity is a sequence of equal payments made at periodic yet equal intervals. In this section you will consider only annuities that have a specific period of time with equal payments and equal payment intervals. If you made equal payments of $30 every month for one year, you would be investing in an annuity of this particular type. An annuity can also pay fixed amounts at regular intervals over a specified period of time. To understand annuities, the meaning of special terms needs to be understood.

Term of an annuity is the length of time from the beginning of the first interval to the end of the last interval.

Payment interval is the length of time between payments.

Periodic payment is the amount of money paid at each interval.

In an ordinary annuity, the periodic payments are made at the end of each time interval.

When working with annuities, you need to know the following:

- the amount to be paid or received in each interval or payment
- the number of equal payments or intervals
- the rate of compound interest
- the total amount of money accumulated through payments and interest

Now do the following exercise.

Define each of the following.

1. annuity
2. simple interest
3. compound interest
4. term of an annuity
5. payment interval of an annuity
6. periodic payment of an annuity

For solutions to **Activity 1**, turn to **Appendix A, Topic 2.**

Activity 2

Develop a repeating process to find the amount and the present value of an annuity.

Consider the following example to help you understand how you calculate the amount of an annuity.

Example 1

The Scober family wants to buy a boat in one year from now. At the end of each month they put \$125.00 into an account which pays 12%/a compounded monthly. How much do they have in the account that was set aside to purchase the boat at year's end?

Solution:

You will remember that 12%/a compounded monthly is 1% per month. The monthly payments can be shown on a time line. Notice that each monthly payment earns interest according to the compound interest formula.

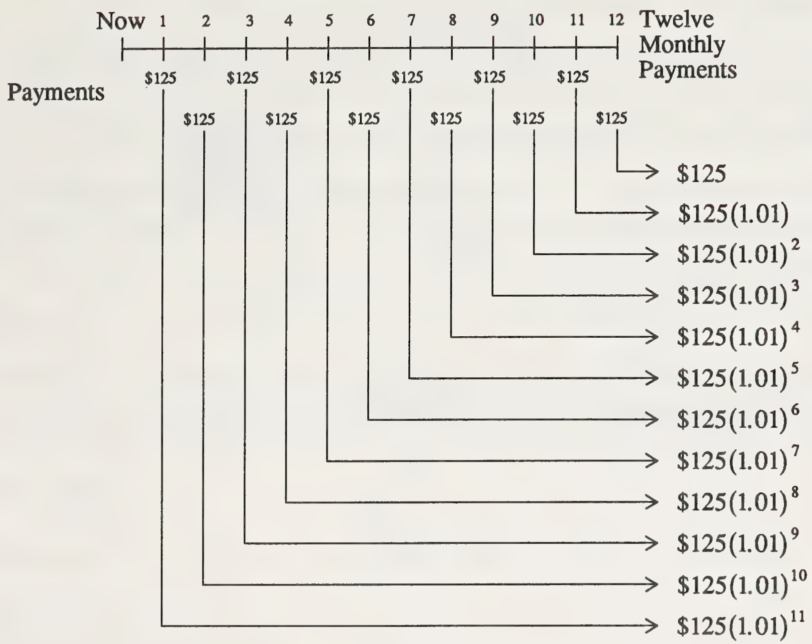
From the time line you will see that

- a regular deposit of \$125 has been made at the end of each month for twelve months
- the payment made in the last month does not earn any interest so the amount is \$125

In this course the payment interval will be the same as the compounding period.

Note: This is an annuity since a fixed amount of \$125 is deposited at the regular interval of once a month over a specified period of time, namely, one year.

Annuity Time Line



The total amount in the account at the end of one year is the sum of the amounts for each period.

Recall: Each payment earns interest that is compounded according to the formula $A = P(1+i)^n$.

$$\begin{aligned}\text{Total} &= \$125(1.01)^{11} + \$125(1.01)^{10} + \$125(1.01)^{9} + \$125(1.01)^{8} + \$125(1.01)^{7} \\ &\quad + \$125(1.01)^{6} + \$125(1.01)^{5} + \$125(1.01)^{4} + \$125(1.01)^{3} + \$125(1.01)^{2} \\ &\quad + \$125(1.01) + \$125 \\ &= \$139.46 + \$138.08 + \$136.71 + \$135.36 + \$134.02 + \$132.69 + \$131.38 \\ &\quad + \$130.08 + \$128.79 + \$127.51 + \$126.25 + \$125.00 \\ &= \$1585.33\end{aligned}$$

The total amount in the account at the end of one year is \$1585.33.

This is a rather lengthy procedure. Later you will see that there are easier and shorter ways to find the amount of an annuity.

The following example shows how the present value of an annuity can be found.

Example 2

The Ishmar family intends to buy a car now. They will need \$345 per month to make the payments for one year. How much must they put into an account now so that there is enough cash available to make the twelve monthly payments? The interest rate is 12%/a compounded monthly.

Solution:

The amount needed now is called the present value of the annuity. When finding compound interest, you used $A = P(1+i)^n$, where P is the initial principal invested or the present value. Dividing both sides by $(1+i)^n$ gives the formula $P = \frac{A}{(1+i)^n}$. This formula is usually written as $PV = \frac{A}{(1+i)^n}$ where PV stands for the present value. This formula is used to calculate the present value of an annuity. Remember that an annuity is a sum of payments, so when you find the present value of an annuity, it will be a sum of present values as this example illustrates.

The term is one year.
The payment interval is one month.
The annual rate of interest is 12%.
The periodic payment is \$125.
The amount of the annuity is \$1585.33.

The principal is the amount that must be invested now to yield a certain amount of money (A) after a certain period of time. Thus, the principal is the present value of the annuity.

Money is invested now so that at the end of each month for one year $345 is available to make the required payment. The first $345 will have one month to earn interest. Thus, the principal or present value required will be less than $345. The required present value can be calculated using the compound interest formula.

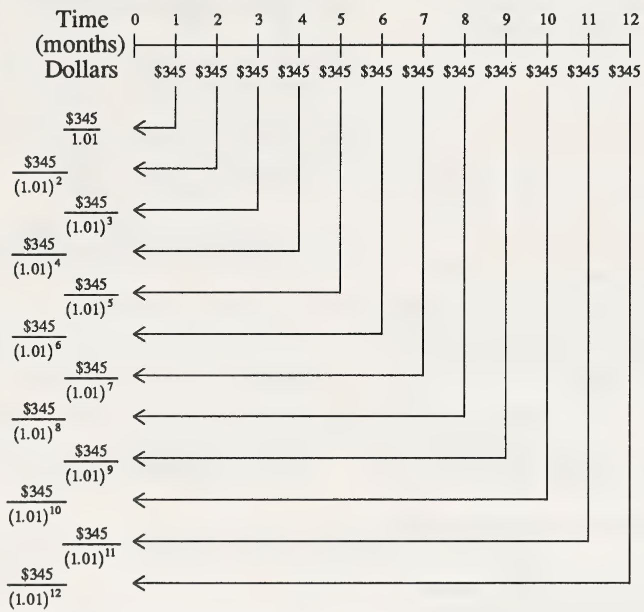
$$A = P(1+i)^n$$

$$\$345 = P(1+0.01)^1$$

$$P = \frac{\$345}{(1.01)}$$

$$= \$341.58$$

The following time line shows the monthly payments needed to pay off the cost of the car.



From the diagram, you can see the following:

- There are twelve monthly payments.
- The annual interest rate is 12%.
- The rate of interest for each monthly compounding period is 1%.
- At the end of each month $345 must be available to make the payment.

$$\text{Present Value} = \frac{\$345}{1.01} + \frac{\$345}{(1.01)^2} + \frac{\$345}{(1.01)^3} + \frac{\$345}{(1.01)^4} + \frac{\$345}{(1.01)^5} + \frac{\$345}{(1.01)^6} + \frac{\$345}{(1.01)^7} + \frac{\$345}{(1.01)^8} + \frac{\$345}{(1.01)^9} + \frac{\$345}{(1.01)^{10}} + \frac{\$345}{(1.01)^{11}} + \frac{\$345}{(1.01)^{12}}$$

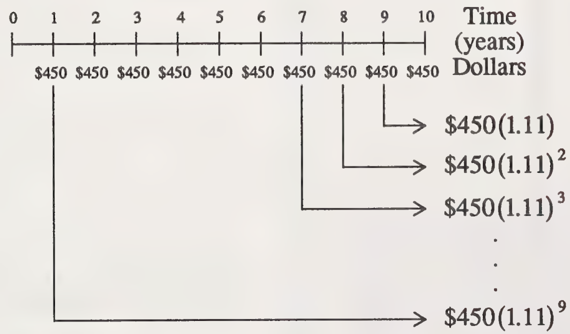
$$\begin{aligned} PV &= \$341.58 + \$338.20 + \$334.85 \\ &+ \$331.54 + \$328.26 + \$325.01 \\ &+ \$321.79 + \$318.60 + \$315.45 \\ &+ \$312.32 + \$309.23 + \$306.17 \\ &= \$3883.00 \end{aligned}$$

The present value is approximately $3883. This represents the amount that must be invested now in order to make twelve monthly payments of $345 over the next year.

Do some practice exercises on your own.

Complete either the odd- or the even-numbered questions.

1. Annuity Time Line



a. What is the term of the annuity?

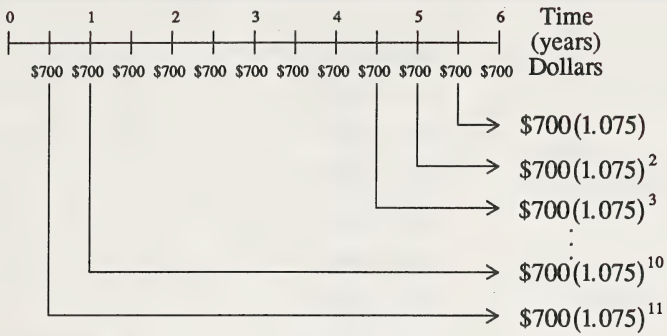
b. What is the annual interest rate?

c. How often is the interest compounded?

d. What is the payment interval? How much is the payment?

e. Find the amount of the annuity.

2. Annuity Time Line



a. What is the term of the annuity?

b. What is the annual interest rate?

c. How often is the interest compounded?

d. What is the payment interval? How much is the payment?

e. Find the amount of the annuity.

3. The Kelsey family won a lottery. How much should they invest now so that they can receive $2500 at the end of each year for the next five years? The interest rate is $9\frac{1}{4}\%$/a compounded annually.

4. What is the present value of an annuity if payments of $950 are made at the end of every six months for eight years? The interest rate is 14%/a compounded semiannually.

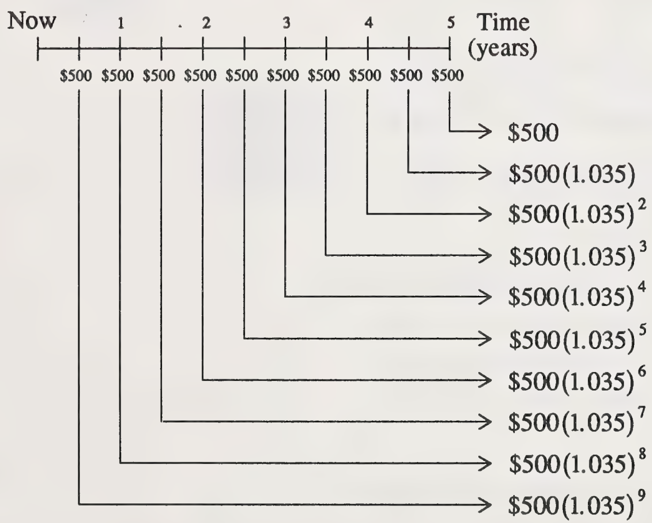
For solutions to **Activity 2,** turn to **Appendix A, Topic 2.**

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
You may decide to do both.

Extra Help

Look at a couple of annuity time lines in more detail to see what information can be derived from them.

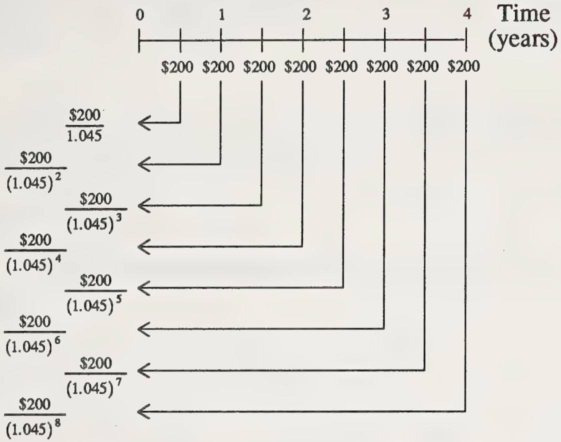
Annuity Time Line (Amount)



- The term of this annuity is five years.
- The annual interest rate is 7%.
- The interest is compounded semiannually.
- The payment interval is every six months or every half year.
- The amount of this annuity is as follows:

$$\begin{aligned} A &= \$500 + \$500(1.035) + \$500(1.035)^2 + \$500(1.035)^3 \\ &\quad + \$500(1.035)^4 + \$500(1.035)^5 + \$500(1.035)^6 \\ &\quad + \$500(1.035)^7 + \$500(1.035)^8 + \$500(1.035)^9 \\ &= \$500.00 + \$517.50 + \$535.61 + \$554.36 + \$573.76 + \$593.84 \\ &\quad + \$614.63 + \$636.14 + \$658.40 + \$681.45 \\ &= \$5865.69 \end{aligned}$$

Annuity Time Line (Present Value)

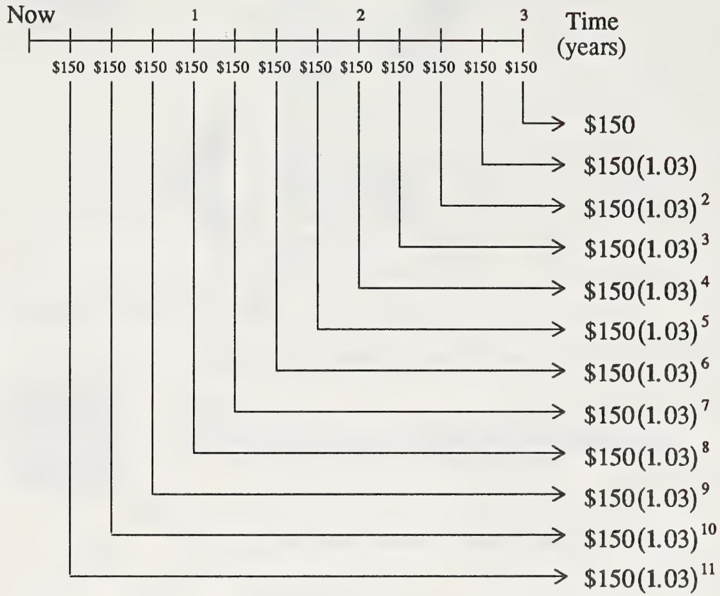


- The term of the annuity is four years.
- There are eight semiannual payments.
- The annual interest rate is 9%.
- The semiannual interest rate is 4.5%.
- The present value (*PV*) of this annuity is as follows:

$$PV = \frac{\$200}{1.045} + \frac{\$200}{(1.045)^2} + \frac{\$200}{(1.045)^3} + \frac{\$200}{(1.045)^4}$$
$$+ \frac{\$200}{(1.045)^5} + \frac{\$200}{(1.045)^6} + \frac{\$200}{(1.045)^7} + \frac{\$200}{(1.045)^8}$$
$$= \$191.39 + \$183.15 + \$175.26 + \$167.71$$
$$+ \$160.49 + \$153.58 + \$146.97 + \$140.64$$
$$= \$1319.19$$

Now try the following exercises.

1. Annuity Time Line (Amount)



Use the time line to answer the following questions.

a. What is the term of this annuity?

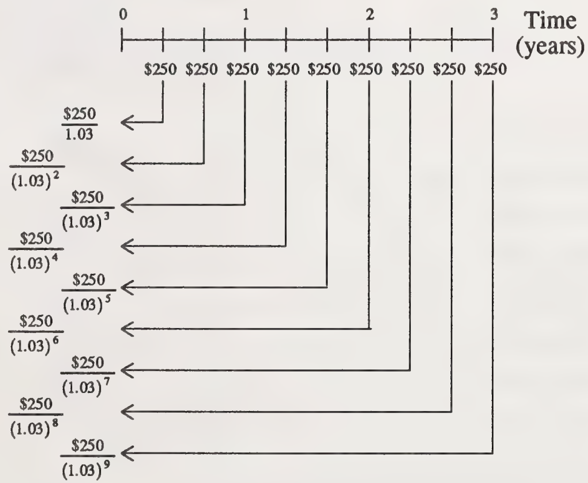
b. What is the annual interest rate?

c. How often is the interest compounded?

d. What is the payment interval? How much is the payment?

e. What is the amount of this annuity?

2. Annuity Time Line (Present Value)



Use the time line to answer the following questions.

a. What is the term of this annuity?

b. What is the annual interest rate?

c. How often is the interest compounded?

d. What is the payment interval? How much is the payment?

e. What is the present value of this annuity?

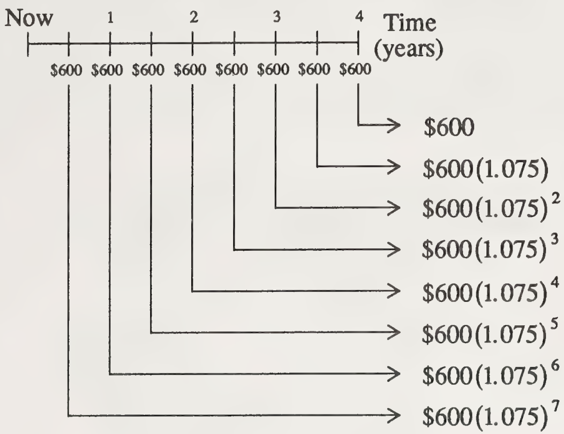
For solutions to **Extra Help**, turn to **Appendix A, Topic 2.**

Extensions

In this section you will find the amount and the present value for the same annuity using a time line in each case.

Amount of an Annuity

Annuity Time Line (Amount)



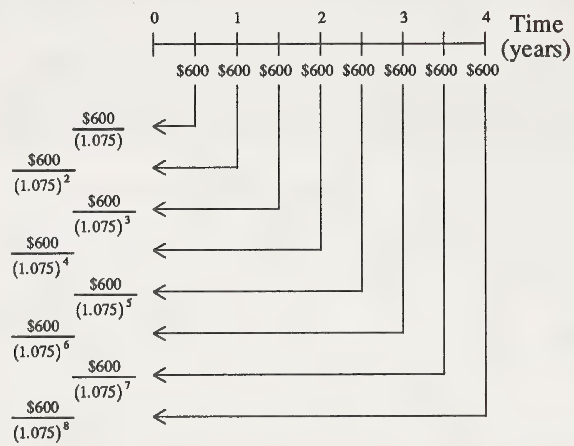
For the previous annuity time line and for the next one, the following is true:

- The term is four years.
- The annual interest rate is 15%.
- The interest is compounded semiannually.
- The payment interval is every six months.
- The amount paid at the end of each interval is $600.

$$\begin{aligned}\text{Amount} &= \$600 + \$600(1.075) + \$600(1.075)^2 \\ &\quad + \$600(1.075)^3 + \$600(1.075)^4 \\ &\quad + \$600(1.075)^5 + \$600(1.075)^6 \\ &\quad + \$600(1.075)^7 \\ &= \$600.00 + \$645.00 + \$693.38 + \$745.38 \\ &\quad + \$801.28 + \$861.38 + \$925.98 + \$995.43 \\ &= \$6267.83\end{aligned}$$

Present Value of an Annuity

Annuity Time Line (Present Value)



$$\text{Present Value} = \frac{\$600}{1.075} + \frac{\$600}{(1.075)^2} + \frac{\$600}{(1.075)^3} + \frac{\$600}{(1.075)^4} + \frac{\$600}{(1.075)^5} + \frac{\$600}{(1.075)^6} + \frac{\$600}{(1.075)^7} + \frac{\$600}{(1.075)^8}$$

$$= \$558.14 + \$519.20 + \$482.98 + \$449.28 + \$417.94 + \$388.78 + \$361.65 + \$336.42$$

$$= \$3514.39$$

In the next topic you will see that tables as well as formulas can be used to find the amount for an annuity and the present value of an annuity.

An advantage of using a time line is that the amount and the present value can be found when tables are not available or when the formulas cannot be remembered.

The disadvantage is that there is so much more work involved when using a time line compared to the use of tables or formulas. This will become apparent in the next topic.

In either case a calculator helps and should therefore be used as much as possible.

Now try the following exercises.

For question 1, make a time line for the amount of the annuity and find the actual amount.

For question 2, make a time line for the present value of the annuity and find the actual present value.

1. At the end of each six-month period for five years, the Goldstein family deposits $475 into an account that pays 11.5%/a compounded semiannually.

2. How much money must the Dubois family set aside now at 9.6%/a, compounded three times per year, in order for them to receive $675 every four months for a period of four years.

For solutions to **Extensions,** turn to **Appendix A, Topic 2.**

Topic 3 Calculating the Amount and the Present Value for Annuities

Introduction

In the last topic you used a time line to find the amount of an annuity and the present value of an annuity. In this topic you will use specially prepared tables and specially developed formulas to calculate the amount and the present value of an annuity.

What Lies Ahead

Throughout the topic you will learn to

1. determine the amount of an annuity by using tables
2. determine the amount of an annuity by applying the formula

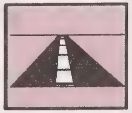
   $$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

3. determine the present value of an annuity by using tables
4. determine the present value of an annuity by applying the formula $PV = \dfrac{R\left[1-(1+i)^{-n}\right]}{i}$
5. solve word problems involving the amount and the present value of an annuity

Now that you know what to expect, turn the page to begin your study of calculating the amount and the present value for annuities.

Exploring Topic 3

Activity 1



Determine the amount of an annuity by using tables.

In this section you will be asked to refer to the specially prepared tables in **Appendix B**.

Instead of using a time line to calculate the amount of an annuity as you did previously, a special table will be used to make your calculations easier and much shorter. Study the following examples to see how this is done. The amount of an annuity table is often referred to as the $s_{\overline{n}|i}$ table. The basic rule used is as follows:

$A = Rs_{\overline{n}|i}$, where

A = amount of the annuity,
R = the periodic payment of the annuity,
n = the number of payments of the annuity,
i = the rate of interest per payment period, and
$s_{\overline{n}|i}$ = the amount of the annuity of \$1 per period at rate i from the table

The payment interval must be the same as the compounding period.

Example 1

The McAlpin family is saving for a holiday. They put \$122.50 per month into an account which pays 12% compounded monthly. How much will they save in two years?

Solution:

Use the table in **Appendix B**.

$A = Rs_{\overline{n}|i}$, where

A = amount saved,
R = periodic payment of \$122.50,
n = number of payments which is 24, and
i = interest rate per period which is 1%

$$A = \$122.50 \times s_{\overline{n}|i}$$

$$= \$122.50 \times 26.973\,46 \quad (s_{\overline{n}|i} = \$1_{\overline{24}|1\%})$$

$$= \$3304.25$$

The amount saved would be \$3304.25 to the nearest cent.

The 26.973 46 value means that \$26.97 would be saved if \$1 were deposited monthly for two years at 12%/a compounded monthly.

To get 26.973 46, go down to 24 in the column headed by n, then go across to the value in the column headed by 1%.

Example 2

Scople and Clapid wanted to know how much more they would save if they deposited \$35.75 every three months into an account which paid 10%/a compounded quarterly for three years compared to depositing \$50.25 every four months into an account which paid 12%/a compounded every four months for two years.

Solution:

Deposit 1

$A = Rs_{\overline{n}|i}$

$= \$35.75 \times 13.795\,55 \; (s_{\overline{n}|i} = \$1_{\overline{12}|2.5\%})$

$= \$493.19$

The amount saved would be \$493.19 (to the nearest cent).

Deposit 2

$A = Rs_{\overline{n}|i}$

$= \$50.25 \times 6.632\,98 \; (s_{\overline{n}|i} = \$1_{\overline{6}|4\%})$

$= \$333.31$

The amount saved would be \$333.31 (to the nearest cent).

Take the difference.

$\$493.19 - \$333.31 = \$159.88$

They would save \$159.88 more in the first annuity compared to the second annuity.

Now try some of the following questions on your own.

Use the tables in **Appendix B** to find the amounts for each of the following annuities. Complete any three problems.

1. Periodic payment is \$325 per month.
   Time period of the annuity is four years.
   Interest rate is 12%/a compounded monthly.

2. Periodic payment is \$1011 every six months.
   Time period of the annuity is ten years.
   Interest rate is 8%/a compounded semiannually.

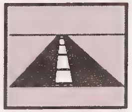
3. Periodic payment is \$3064.15 every three months.
   Time period of the annuity is five years.
   Interest rate is 10%/a compounded quarterly.

4. Periodic payment is \$75.49 every four months.
   Time period of the annuity is seven years.
   Interest rate is 15%/a compounded every four months.

5. Periodic payment is \$2004.16 twelve times a year.
   Time period of the annuity is four years.
   Interest rate is 18%/a compounded monthly.

For solutions to **Activity 1,** turn to **Appendix A, Topic 3.**

Activity 2



Determine the amount of an annuity by applying the formula $A = \frac{R\left[(1+i)^n - 1\right]}{i}$.

Sometimes annuity tables may not be available. In such cases a specially developed formula can be used. The formula is also useful when the given values are not found in the tables.

The formula is $A = \frac{R\left[(1+i)^n - 1\right]}{i}$, where

A = the amount of the annuity,
R = the periodic payment of the annuity,
i = the interest rate per period, and
n = the number of interest periods.

In this formula the payment interval must be the same as the compounding period.

You are encouraged to use your calculator to find solutions. For each example that follows, the calculator procedure is outlined for you.

Example 3

Find the amount of an annuity for which \$248.50 is deposited every half year for fifteen years at a rate of $10\frac{1}{2}\%$/a compounded semiannually.

Solution:

Use $A = \frac{R\left[(1+i)^n - 1\right]}{i}$, where

$$R = \$248.50,$$
$$i = \frac{1}{2} \times 0.105 = 0.0525, \text{ and}$$
$$n = 15 \times 2 = 30.$$

$$A = \frac{\$248.50\left[(1+0.0525)^{30} - 1\right]}{0.0525}$$
$$= \frac{\$248.50 \times 3.641\,551\,091}{0.0525}$$
$$= \$17\,236.68 \text{ (to the nearest cent)}$$

Use your calculator to solve this equation.

Enter	Display
C	0
1.0525	1.0525
x^y	1.0525
30	30
−	4.641551091
1	1
=	3.641551091
×	3.641551091
248.50	248.50
÷	904.925446
0.0525	0.0525
=	17236.67516

Example 4

Find the amount of an annuity for which $1032 is deposited every two months for seven years. The interest rate is 12%/a compounded every two months.

Solution:

Use $A = \dfrac{R\left[(1+i)^n - 1\right]}{i}$, where

$$R = \$1032,$$

$$i = \frac{1}{6} \times 0.12$$

$$= 0.02, \text{ and}$$

$$n = 7 \times 6$$

$$= 42.$$

$$A = \frac{\$1032\left[(1+0.02)^{42} - 1\right]}{0.02}$$

$$= \frac{\$1032 \times 1.297\,244\,466}{0.02}$$

$$= \$66\,937.81 \text{ (to the nearest cent)}$$

Check your answer by using the table.

$$A = Rs_{\overline{n}|i}$$

$$A = \$1032 \times 64.862\,22$$

$$A = \$66\,937.81 \text{ (to the nearest cent)}$$

Now do any three of the following questions.

Use the tables in **Appendix B** and the formula for the amount of an annuity to find the amount of the annuity for each of the following. Your answers should be the same for both methods.

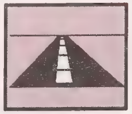
1. $350.22 deposited every six months for five years at 6%/a compounded semiannually
2. $916.14 deposited every three months for seven years at 16%/a compounded quarterly
3. $1160.15 deposited every year for twenty years at 9%/a compounded yearly
4. $955.60 deposited every month for three years at 18%/a compounded monthly
5. $2007.10 deposited every four months for ten years at 15%/a compounded three times a year

For solutions to **Activity 2**, turn to **Appendix A, Topic 3.**

Use your calculator to solve this equation.

Enter	Display
C	0
1.02	1.02
x^y	1.02
42	42
−	2.297244466
1	1
=	1.297244466
×	1.297244466
1032	1032
÷	1338.756289
0.02	0.02
=	66937.81445

Activity 3



Determine the present value of an annuity by using tables.

In this section you will take advantage of still other specially prepared tables found in **Appendix B**.

In a previous section you saw how a time line can be used to find the present value of an annuity. This was a lot of work since each period had to be calculated separately. When all periods were calculated, the resulting values were added to determine the present value of the annuity. There is an easier way. Study the following examples to see how the present-value table is used. The present-value table is often referred to as the $Ra_{\overline{n}|i}$ table. The basic rule used is as follows:

$PV = Ra_{\overline{n}|i}$, where

PV = the present value of the annuity,
R = the periodic payment of the annuity,
n = the number of payments of the annuity,
i = the rate of interest per payment period, and
$a_{\overline{n}|i}$ = the present value of an annuity of \$1 per period at rate i

The payment interval must be the same as the compounding period.

Example 5

Find the present value of an annuity that has payments of \$350 made at the end of six-month periods for fifteen years at 10%/a compounded semiannually.

Solution:

Use the table in **Appendix B** and the following formula.

$PV = Ra_{\overline{n}|i}$, where

PV = the present value of the annuity,
R = \$350 (periodic payment),
n = 30 (number of payments),
i = 5% (interest rate per period), and
$a_{\overline{n}|i}$ = 15.372 45 (present value on \$1)

$$PV = \$350 \times 15.372\,45$$
$$= \$5380.36$$

The present value for this annuity is \$5380.36.

This means that you would have to invest \$5380.36 now in order to provide semiannual payments of \$350 for fifteen years if interest is at 10%/a compounded semiannually.

Note: To find the value for $a_{\overline{n}|i}$, locate 30 in column n and go across to the value found in the column headed by 5%.

$$PV = Ra_{\overline{n}|i}$$

Present value
Regular payment
Number of payments
Interest rate per period

Example 6

The Mielke family is the beneficiary of a will. The will specifies that the Mielkes will receive $1500 every three months for six years. The interest is compounded quarterly at 10%/a. What sum of money was needed initially to set up such an annuity?

Solution:

$$
\begin{aligned}
PV &= Ra_{\overline{n}|i} \\
&= \$1500 \times 17.884\,99 \\
&= \$26\,827.49
\end{aligned}
$$

The initial sum of money invested to set up this annuity was $26 827.49.

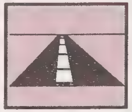
Do any three of the following problems.

Use the table in **Appendix B** to find the present value for each of the following annuities.

1. $350.62 paid out every six months for five years at 6%/a compounded semiannually
2. $916.14 paid out every three months for seven years at 16%/a compounded quarterly
3. $1160.15 paid out every year for twenty years at 9%/a compounded yearly
4. $955.60 paid out every month for three years at 18%/a compounded monthly
5. $2007.10 paid out every four months for ten years at 15%/a compounded three times a year

For solutions to **Activity 3,** turn to **Appendix A, Topic 3.**

Activity 4



Determine the present value of an annuity by applying the formula $PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$.

As with the amount of an annuity, tables may not always be available to find the present value of an annuity. If this is the situation, the following formula can be used to find the present value for any annuity.

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}, \text{ where}$$

PV = present value,
R = the periodic payment of the annuity,
i = interest rate per period, and
n = number of interest periods

You are encouraged to use a calculator when using this formula to find the present value of an annuity. For each example that follows, the calculator procedure will be outlined for you.

Example 7

What is the present value of an annuity that has payments of $175 made at the end of every six months for fifteen years compounded semiannually at 10%/a?

Solution:

Use $PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$, where

$$\begin{aligned} R &= \$175 \\ i &= \frac{1}{2}\times 0.10 \\ &= 0.05, \text{ and} \\ n &= 15\times 2 \\ &= 30. \end{aligned}$$

$$\begin{aligned} PV &= \frac{\$175\left[1-(1+0.05)^{-30}\right]}{0.05} \\ &= \$2690.178\,93 \\ &= \$2690.18 \text{ (to the nearest cent)} \end{aligned}$$

The present value for this annuity is $2690.18.

Use your calculator to solve this equation.

Enter	Display
C	0
1	1
−	1
1.05	1.05
x^y	1.05
30	30
+/−	−30
=	0.768622551
×	0.768622551
175	175
÷	134.5089465
0.05	0.05
=	2690.17893

Example 8

The Shimbashi family plans to invest some money now so that they can receive $1000 at the end of each year for the next three years. If the money is invested at $7\frac{1}{2}\%/a$ compounded annually, how much must the Shimbashi family invest?

Solution:

Use $PV = \dfrac{R\left[1-(1+i)^{-n}\right]}{i}$, where

$R = 1000$,

$i = 0.075$, and

$n = 3$.

$$PV = \frac{\$1000\left[1-(1+0.075)^{-3}\right]}{0.075}$$

$$= \$2600.525\,739$$

$$= \$2600.53 \text{ (to the nearest cent)}$$

The Shimbashi family must invest $2600.53 now.

Do any three of the following five questions.

Using the present-value formula, find the present value for each of the following.

1. $350.62 paid out every six months for five years at 6%/a compounded semiannually
2. $916.14 paid out every three months for seven years at 16%/a compounded quarterly
3. $1160.15 paid out every year for twenty years at 9%/a compounded yearly
4. $955.60 paid out every month for three years at 18%/a compounded monthly
5. $2007.10 paid out every four months for ten years at 15%/a compounded three times a year

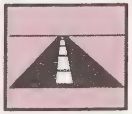
Did you get the same answers for these problems as you did for the Activity 3 problems? If not, check to see why not, since they should be the same or close to being the same.

For solutions to **Activity 4**, turn to **Appendix A, Topic 3.**

Use your calculator to solve this equation.

Enter	Display
C	0
1	1
−	1
1.075	1.075
x^y	1.075
3	3
+/−	−3
=	0.19503943
×	0.19503943
1000	1000
÷	195.0394304
0.075	0.075
=	2600.525739

Activity 5



Solve word problems involving the amount and the present value of an annuity.

When solving word problems related to annuities, you must decide whether the situation deals with the amount of an annuity or the present value of an annuity. Once this decision is made, you can use the correct annuity table or the appropriate formula to find the solution. In the examples that follow, the solutions will be developed using the tables and the formulas.

Example 9

The Perez family decided that in the future they would like to add a swimming pool to their backyard. This project would require a lot of money, so they decided to invest $250 every three months in an annuity which pays 12%/a compounded quarterly. How much money will they have for this project in five years?

Solution:

You are asked to find the amount of the annuity.

Using the table method, you get the following:

$$\begin{aligned} A &= Rs_{\overline{n}|i} \\ &= \$250 \times 26.870\,37 \\ &= \$6717.5925 \\ &= \$6717.59 \end{aligned}$$

The total amount available after five years is $6717.59.

Using the formula method, you get the following:

$$\begin{aligned} A &= \frac{R\left[(1+i)^n - 1\right]}{i} \\ &= \frac{\$250\left[(1+0.03)^{20} - 1\right]}{0.03} \\ &= \frac{\$250\left[(1.03)^{20} - 1\right]}{0.03} \\ &= \$6717.593\,622 \\ &= \$6717.59 \end{aligned}$$

The total amount available after five years is $6717.59.

To get 26.870 37, find 20 in column n, and then go across to the column headed by 3%. This value is located at the point where this row and this column intersect.

Periodic payment = $250

$$\begin{aligned} \text{Number of periods} &= 5 \times 4 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{Interest rate} &= \frac{12\%}{4} \\ &= 3\% \end{aligned}$$

Use your calculator to solve this equation.

Enter	Display
C	0
1.03	1.03
x^y	1.03
20	20
−	1.806111235
1	1
=	0.806111235
×	0.806111235
250	250
÷	201.5278087
0.03	0.03
=	6717.593622

Both methods result in the same answer.

Example 10

A group of employees at an auto plant won a lottery. Employee #2010134 wants to use the lottery prize to establish an annuity which would pay out $500 every three months for the next twelve years. If the money is worth 12%/a compounded quarterly, then how much does it take to set up this annuity?

Solution:

You are asked to find the present value of this annuity.

Using the table method, you get the following:

$$\begin{aligned} PV &= Ra_{\overline{n}|i} \\ &= \$500 \times 25.266\,71 \\ &= \$12\,633.355 \\ &= \$12\,633.36 \end{aligned}$$

It would take $12 633.36 to set up this annuity.

Using the formula method, you get the following:

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$= \frac{\$500\left[1-(1+0.03)^{-48}\right]}{0.03}$$

$$= \$12\,633.353\,32$$

$$= \$12\,633.35$$

It would take $12 633.35 to set up this annuity.

Use your calculator to solve this equation.

Enter	Display
C	0
1	1
−	1
1.03	1.03
x^y	1.03
48	48
+/−	−48
=	0.758001199
×	0.758001199
500	500
÷	379.0005995
0.03	0.03
=	12633.35332

PV = present value
R = periodic payment
= $500
n = number of payments
= 48
i = interest rate per payment period
= 3%
$a_{\overline{n}|i}$ = amount paid out on a $1 investment under these specific conditions
= $\$1_{\overline{48}|3\%}$

Try some similar problems on your own.

Do any five of the following questions. Use the table and formula methods to find the answers. Use a calculator.

1. A farmer wishes to establish a fund to replace his combine in five years. If he deposits $750 every six months into an account which pays interest at 10%/a compounded semiannually, how much cash will be available at the end of the five-year time period?

2. An individual deposits $525 at the end of each three-month period into a fund which credits interest at 10%/a compounded quarterly. Find, to the nearest dollar, the sum of money available at the end of the sixth year.

3. To buy a new car, the Volcek family must pay $450 per month for four years. Interest is charged at 18%/a compounded monthly. Find the amount the Volceks would pay if they paid cash instead of going through a finance company.

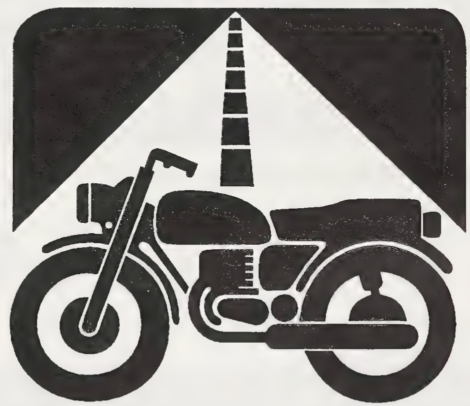
4. How much must be invested now for an annuity to pay $750 every six months for ten years at 8%/a compounded semiannually?

5. An annuity of $1500 is paid every four months for six years. Interest is compounded every four months at 9%/a. How much is in the annuity at the present time?

6. Desirée and Jean-Paul are fourteen-year-old twins. They babysit and plan to get part-time jobs in order to pay for part of their postsecondary education. They intend to save $85 every month for the next four years. How much money will they have if this money is invested in an account which pays 12%/a compounded monthly?

7. To pay for a fancy motorbike, Rosa and Pedro have to make monthly payments of $149 for $3\frac{3}{4}$ years. If interest is compounded monthly at 18%/a, what would they have paid for the bike if this was a cash purchase? How much would the credit payments amount to? How much would they save by paying cash?

For solutions to **Activity 5,** turn to **Appendix A, Topic 3.**



If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

You may decide to do both.

Extra Help

The easiest way to find the amount and the present value of an annuity is to use the specially prepared tables. These tables are found in **Appendix B** of this unit.

To calculate the amount, you must know the following:

- periodic payment
- annual interest rate
- compounding period
- interest rate per compounding period

Example 11

Use the table in **Appendix B** to calculate the amount for an annuity for which \$350 is deposited every month into an account which pays 6%/a interest compounded monthly for three years.

Solution:

Periodic payment = \$350
Annual interest rate = 6%
The compounding period is once every month or twelve times a year or thirty-six times for three years.

The interest rate per compounding period is

$6\% \div 12 = 0.50\%$ or $\frac{1}{2}\%$.

Amount of Annuity Table

n	$\frac{1}{2}\%$	1%	$1\frac{1}{2}\%$	2%	$2\frac{1}{2}\%$	3%
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.00500	2.01000	2.01500	2.02000	2.02500	2.03000
3	3.01503	3.03010	3.04523	3.06040	3.07563	3.09090
4	4.03010	4.06040	4.09090	4.12161	4.15252	4.18363
5	5.05025	5.10101	5.15227	5.20404	5.25633	5.30914
⋮	⋮	⋮	⋮	⋮	⋮	⋮
26	27.69191	29.52563	31.51397	33.67091	36.01171	38.55304
27	28.83037	30.82089	32.98668	35.34432	37.91200	40.70963
28	29.97452	32.12910	34.48148	37.05121	39.85980	42.93092
29	31.12439	33.45039	35.99870	38.79223	41.85630	45.21885
30	32.28002	34.78489	37.53868	40.56808	43.90270	47.57546
31	33.44142	36.13274	39.10176	42.37944	46.00027	50.00268
32	34.60862	37.49408	40.68829	44.22703	48.15028	52.50276
33	35.78167	38.86901	42.29861	46.11157	50.35403	55.07784
34	36.96058	40.25770	43.93309	48.03380	52.61289	57.73018
35	38.14538	41.66028	45.59209	49.99448	54.92821	60.46208
36	39.33611	43.07688	47.27597	51.99437	57.30141	63.27594
37	40.53279	44.50765	48.98511	54.03425	59.73395	66.17422
38	41.73545	45.95272	50.71989	56.11494	62.22730	69.15945
39	42.94413	47.41225	52.48068	58.23724	64.78298	72.23423
40	44.15885	48.88637	54.26789	60.40198	67.40255	75.40126

The 39.336 11 would be the amount if \$1 were to be deposited every month for three years at 6%/a compounded monthly.

In $A = Rs_{\overline{n}|i}$ and using \$1,

$A = 1_{\overline{36}|\frac{1}{2}\%}$ or \$39.336 11.

$$A = Rs_{\overline{n}|i}$$
$$= \$350 \times 39.336\,11$$
$$= \$13\,767.638\,5$$
$$= \$13\,767.64$$

The annuity would amount to \$13 767.64 at the end of three years.

Once the n-value exceeds 50, you would have to use the amount formula.

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

The formula is also used when the specified interest rate is not provided in the table. Some examples would be $\frac{3}{4}\%$, $2\frac{7}{8}\%$, ...

To calculate present value, the procedure would be exactly the same as for finding the amount of an annuity except that a different table would be used.

Example 12

Use the table in **Appendix B** to calculate the present value of an annuity where \$350 is paid out every month from an account which pays 6%/a interest compounded monthly for three years.

Solution:

$$R = \$350$$
$$n = 3 \times 12$$
$$= 36$$
$$i = 6\% \div 12$$
$$= \frac{1}{2}\%$$

$$PV = Ra_{\overline{n}|i}$$
$$= \$350 \times 32.871\,02$$
$$= \$11\,504.857$$
$$= \$11\,504.86$$

The present value of this annuity is \$11 504.86.

Try the following questions for more practice. Use the tables in **Appendix B**.

1. Find the amount of each of the following annuities.

   a. \$550 deposited every three months for seven years at 10%/a compounded quarterly

   b. \$1034.16 deposited every six months for twelve years at 14%/a compounded semiannually

   c. \$139.43 deposited every month for four years at 18%/a compounded monthly

2. Find the present value of each of the following annuities.

   a. \$75 paid out every four months for five years at 15%/a compounded every third of a year

   b. \$2003.15 paid out every year for six years at 7%/a compounded yearly

   c. \$511 paid out every month for $2\frac{1}{2}$ years at 18%/a compounded monthly

For solutions to **Extra Help,** turn to **Appendix A, Topic 3.**

Extensions

Most of the situations in this unit have involved only single investments. This does not have to be the case. Study the next example.

Example 13

The Olaufson family intends to buy a car five years from now. They have set up two accounts that are outlined as follows.

- In Account #1, they deposit \$50 each month at 12%/a compounded monthly.
- In Account #2, they deposit \$250 every six months at 14%/a compounded semiannually.

In all, how much would the Olaufsons save at the end of the five-year period? Use the amount formula where needed.

Solution:

In Account #1,

$$A=\frac{R\left[(1+i)^n-1\right]}{i}$$

$$A=\frac{\$50\left[(1+0.01)^{60}-1\right]}{0.01}$$

$$A=\$4083.483\,493$$

$$A=\$4083.48$$

The amount in Account #1 at the end of five years would be \$4083.48.

In Account #2,

$$A=Rs_{\overline{n}|i}$$

$$A=\$250\times 13.816\,45$$

$$A=\$3454.1125$$

$$A=\$3454.11$$

The amount in Account #2 at the end of five years would be \$3454.11.

The total amount saved at the end of five years would be $\$4083.48+\$3454.11=\$7537.59$.

In Account #1,

$$R=\$50$$
$$i=\frac{0.12}{12}$$
$$=0.01 \text{ or } 1\%$$
$$n=5\times 12$$
$$=60$$

In Account #2,

$$R=\$250$$
$$n=5\times 2$$
$$=10$$
$$i=14\%\div 2$$
$$=7\%$$

$s_{\overline{n}|i}=\$13.816\,45$ (the amount of the annuity if the amount deposited would be \$1 and the other conditions were maintained)

The formula used to find the amount of an annuity also can be used to find the periodic payment. Look at the next example.

Example 14

Maziar is planning ahead and feels he would like to buy a computer and a laser printer for $8500 six years from now. How much must he put into an account at the end of every month to realize his dream if the bank pays 9%/a compounded monthly?

Solution:

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$\$8500 = \frac{R\left[(1+0.0075)^{72} - 1\right]}{0.0075}$$

$$\$8500 = \frac{R \times 0.712\,552\,706}{0.0075}$$

$$R \times 0.712\,552\,706 = \$8500 \times 0.0075$$

$$0.712\,552\,706R = \$63.75$$

$$\frac{0.712\,552\,706R}{0.712\,552\,706} = \frac{\$63.75}{0.712\,552\,706}$$

$$R = \$89.467\,066\,03$$

$$R = \$89.47$$

$$A = \$8500$$
$$n = 6 \times 12$$
$$= 72$$
$$i = 9\% \div 12$$
$$= 0.0075$$
$$R = ?$$

The amount Maziar would have to deposit each month is $89.47.

In Example 14, a periodic deposit of $89.46 under these conditions would amount to $8499.33 after six years. A deposit of $89.47 would amount to $8500.28. No dollar figure will work out exactly to $8500.00.

If you were to use $89.467 066 03 as a periodic payment, the amount would be $8500.000 009.

Complete the following questions for additional practice.

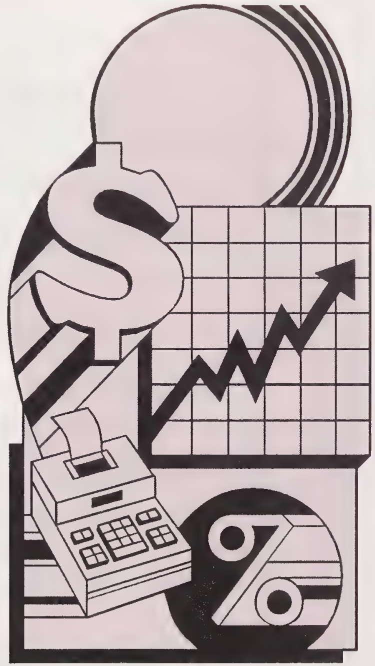
1. Monica wants to buy a stereo system in $2\frac{1}{2}$ years. The system will cost $2200. How much should she deposit at the end of each quarter year if the account she chooses pays 12%/a compounded quarterly? Calculate your answer using the amount formula. Check your answer by using the amount formula again and use the value you calculated for R to see if A is close to $2200.

2. Rupert is hoping to buy a used car four years from now. He finds that he has a choice between two investment plans.
   In Plan A, he would deposit $600 every six months at 11%/a compounded semiannually.
   In Plan B, he would deposit $300 every three months at 10%/a compounded quarterly.
   How much would he have if he chose Plan A? How much would he have if he chose Plan B? What is the difference in the two amounts?

3. Igora invests $75 every four months for five years in an account which pays 12%/a compounded every four months. Redeana invests $45 every month for five years in an account which pays 9%/a compounded monthly. Which of these two people has the most money in her account after the five years? What is the difference in the two amounts?

4. a. What periodic payment at $11\frac{1}{2}$%/a compounded annually and made at the end of each year for 25 years will amount to $20 000? Check your answer using the amount formula a second time and use the value for *R* which was calculated. Is *A* close to being $20 000?

   b. If payments could be made every six months at $11\frac{1}{2}$%/a compounded semiannually, what would each payment be? Check your answer using the amount formula a second time and use the value for *R* which was calculated. Is *A* close to being $20 000?



For solutions to **Extensions,** turn to **Appendix A, Topic 3.**

Unit Summary

What You Have Learned

Having completed this unit, you should be able to do the following:

- Distinguish between simple interest and compound interest.
- Determine the amount of an investment earning simple interest.
- Determine the amount of an investment earning compound interest.
- Recognize an investment which is an annuity.
- Understand the meaning of the amount of an annuity.
- Understand the meaning of the present value of an annuity.
- Apply the compound interest formula to determine the amount of an annuity.
- Determine the present value of an annuity by calculating the sum of the present values of the individual payments.

Unit Summary

- Use the formula $A = Rs_{\overline{n}|i}$ together with the $s_{\overline{n}|i}$ tables to determine the amount of an annuity.
- Apply the formula $A = \dfrac{R\left[(1+i)^n - 1\right]}{i}$ to determine the amount of an annuity.
- Use the formula $PV = Ra_{\overline{n}|i}$ together with the $a_{\overline{n}|i}$ tables to determine the present value of an annuity.
- Apply the formula $PV = \dfrac{R\left[1-(1+i)^{-n}\right]}{i}$ to determine the present value of an annuity.

You are now ready to

complete the **Unit Assignment**.

Appendices

Appendix A
Solutions

Review

Topic 1 Simple Interest and Compound Interest

Topic 2 Introducing Annuities

Topic 3 Calculating the Amount and the Present Value for Annuities

Appendix B
Tables

Table 1 Amount of an Annuity

$A = Rs_{\overline{n}|i}$

Table 2 Present Value of an Annuity

$PV = Ra_{\overline{n}|i}$

Appendix A
Solutions

Review

1. a. $5\% = \frac{5}{100}$
   $= 0.05$

   b. $13\% = \frac{13}{100}$
   $= 0.13$

   c. $10\frac{3}{4}\% = 10.75\%$
   $= \frac{10.75}{100}$
   $= 0.1075$

   d. $\frac{2}{3}\% = 0.\overline{6}\%$
   $= \frac{0.\overline{6}}{100}$
   $= 0.00\overline{6}$ or $0.0066\ldots$

   e. $\frac{7}{20}\% = 0.35\%$
   $= \frac{0.35}{100}$
   $= 0.0035$

   f. $0.3\% = \frac{0.3}{100}$
   $= 0.003$

   g. $0.415\% = \frac{0.415}{100}$
   $= 0.00415$

2. a.

Enter	Display
C	0
0.7	0.7
+/–	– 0.7

b.

Enter	Display
C	0
4.33	4.33
+/–	– 4.33

c.

Enter	Display
C	0
306.44	306.44
+/–	– 306.44

3. a.

Enter	Display
$\boxed{C}$	0
1.23	1.23
$\boxed{x^y}$	1.23
6	6
$\boxed{=}$	3.462825992

$(1.23)^6 \doteq 3.4628$

b.

Enter	Display
$\boxed{C}$	0
1.03	1.03
$\boxed{x^y}$	1.03
22	22
$\boxed{\times}$	1.916103409
1500	1500
$\boxed{=}$	2874.155113

$1500(1.03)^{22} \doteq 2874.1551$

c.

Enter	Display
$\boxed{C}$	0
1.44	1.44
$\boxed{x^y}$	1.44
4	4
$\boxed{+/-}$	-4
$\boxed{=}$	0.232568039

$(1.44)^{-4} \doteq 0.2326$

d.

Enter	Display
$\boxed{C}$	0
1.36	1.36
$\boxed{x^y}$	1.36
5	5
$\boxed{+/-}$	-5
$\boxed{\times}$	0.214934166
1250	1250
$\boxed{=}$	268.6677085

$1250(1.36)^{-5} \doteq 268.6677$

4. a. 406.735 28 rounded to the nearest

   - whole number is 407
   - tenth is 406.7
   - hundredth is 406.74
   - thousandth is 406.735
   - ten thousandth is 406.7353

   b. 3342.063 95 rounded to the nearest

   - whole number is 3342
   - tenth is 3342.1
   - hundredth is 3342.06
   - thousandth is 3342.064
   - ten thousandth is 3342.0640

Exploring Topic 1

Activity 1

Compare the growth of an investment over time using simple and compound interest.

1. $I = Prt$

$= \$6500 \times 0.0925 \times 1$

$= \$601.25$

The interest is \$601.25.

$A = P + I$

$= \$6500.00 + \$601.25$

$= \$7101.25$

The amount is \$7101.25.

2. $I = Prt$

$= \$10\,500 \times 0.0875 \times 1$

$= \$918.75$

The interest is \$918.75.

$A = P + I$

$= \$10\,500.00 + \$918.75$

$= \$11\,418.75$

The amount is \$11 418.75.

3. $I = Prt$

$= \$4550 \times 0.1250 \times 4$

$= \$2275$

The interest is \$2275.

$A = P + I$

$= \$4550 + \$2275$

$= \$6825$

The amount is \$6825.

4. $I = Prt$

$= \$20\,150 \times 0.112 \times 3$

$= \$6770.40$

The interest is $6770.40.

$A = P + I$

$= \$20\,150.00 + \$6770.40$

$= \$26\,920.40$

The amount is $26 920.40.

5. i. $I = Prt$

$= \$1500 \times 0.135 \times 0.25$

$= \$50.63$

$A = P + I$

$= \$1500.00 + \$50.63$

$= \$1550.63$

ii. $I = Prt$

$= \$1550.63 \times 0.135 \times 0.25$

$= \$52.33$

$A = P + I$

$= \$1550.63 + \$52.33$

$= \$1602.96$

iii. $I = Prt$

$= \$1602.96 \times 0.135 \times 0.25$

$= \$54.10$

$A = P + I$

$= \$1602.96 + \$54.10$

$= \$1657.06$

iv. $I = Prt$

$= \$1657.06 \times 0.135 \times 0.25$

$= \$55.93$

$A = P + I$

$= \$1657.06 + \$55.93$

$= \$1712.99$

The amount at the end of one year would be $1712.99.

6. i. $I = Prt$

$= \$1500 \times 0.128 \times 0.5$

$= \$96$

$A = P + I$

$= \$1500 + \$96$

$= \$1596$

ii. $I = Prt$

$= \$1596 \times 0.128 \times 0.5$

$= \$102.14$

$A = P + I$

$= \$1596.00 + \$102.14$

$= \$1698.14$

iii. $I = Prt$

$= \$1698.14 \times 0.128 \times 0.5$

$= \$108.68$

$A = P + I$

$= \$1698.14 + \$108.68$

$= \$1806.82$

The amount at the end of $1\frac{1}{2}$ years would be \$1806.82.

7. a. $A = P(1+i)^n$

$= \$1550(1+0.0475)^6$

$= \$1550(1.0475)^6$

$= \$1550 \times 1.321\,065\,01$

$= \$2047.650\,765$

$= \$2047.65$

$I = A - P$

$= \$2047.65 - \$1550.00$

$= \$497.65$

b. $A = P(1+i)^n$

$= \$8335(1+0.028\,125)^{20}$

$= \$8335(1.028\,125)^{20}$

$= \$8335 \times 1.741\,479\,605$

$= \$14\,515.232\,51$

$= \$14\,515.23$

$I = A - P$

$= \$14\,515.23 - \$8335.00$

$= \$6180.23$

c. In the months of July, August, and September, there is a total of 92 days.

The annual interest rate is $13\frac{3}{4}\%$ or 13.75%. The interest rate per day is $\frac{13.75\%}{365} = \frac{0.1375}{365} = 0.000\,376\,712$.

$A = P(1+i)^n$

$= \$935.18(1+0.000\,376\,712)^{92}$

$= \$935.18(1.000\,376\,712)^{92}$

$= \$935.18(1.035\,258\,346)$

$= \$968.1529$

$= \$968.15$

$I = A - P$

$= \$968.15 - \$935.18$

$= \$32.97$

d. $A = P(1+i)^n$

$$\begin{aligned} &= \$10\,300(1+0.007\,083\,333)^{48} \\ &= \$10\,300(1.007\,083\,333)^{48} \\ &= \$10\,300 \times 1.403\,264\,753 \\ &= \$14\,453.626\,95 \\ &= \$14\,453.63 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$14\,453.63 - \$10\,300.00 \\ &= \$4153.63 \end{aligned}$$

e. $A = P(1+i)^n$

$$\begin{aligned} &= \$7432(1+0.018)^{30} \\ &= \$7432(1.018)^{30} \\ &= \$7432 \times 1.707\,785\,571 \\ &= \$12\,692.262\,37 \\ &= \$12\,692.26 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$12\,692.26 - \$7432.00 \\ &= \$5260.26 \end{aligned}$$

f. $A = P(1+i)^n$

$$\begin{aligned} &= \$125\,000(1+0.019\,583\,33)^{11} \\ &= \$125\,000(1.019\,583\,33)^{11} \\ &= \$125\,000 \times 1.237\,798\,648 \\ &= \$154\,724.831 \\ &= \$154\,724.83 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$154\,724.83 - \$125\,000.00 \\ &= \$29\,724.83 \end{aligned}$$

g. $A = P(1+i)^n$

$$\begin{aligned} &= \$525.75(1+0.061\,25)^{35} \\ &= \$525.75(1.061\,25)^{35} \\ &= \$525.75 \times 8.009\,762\,038 \\ &= \$4211.132\,391 \\ &= \$4211.13 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$4211.13 - \$525.75 \\ &= \$3685.38 \end{aligned}$$

8. a. $A = P(1+i)^n$

$= \$5000(1+0.055)^8$

$= \$5000(1.055)^8$

$= \$5000(1.534\,686\,515)$

$= \$7673.432\,575$

$= \$7673.43$

The accumulated amount is \$7673.43.

b. $A = P(1+i)^n$

$= \$5000(1+0.0275)^{16}$

$= \$5000(1.0275)^{16}$

$= \$5000(1.543\,509\,436)$

$= \$7717.547\,179$

$= \$7717.55$

The accumulated amount is \$7717.55.

9. Bank A

$A = P(1+i)^n$

$= \$3500(1+0.038\,75)^{12}$

$= \$3500(1.038\,75)^{12}$

$= \$3500(1.578\,092\,448)$

$= \$5523.323\,568$

$= \$5523.32$

Bank B

$A = P(1+i)^n$

$= \$3500(1+0.08)^6$

$= \$3500(1.08)^6$

$= \$3500(1.586\,874\,323)$

$= \$5554.060\,13$

$= \$5554.06$

The difference is \$5554.06 – \$5523.32 = \$30.74.

Extra Help

1. a. $I = Prt$

$= \$5600 \times 0.0925 \times 6$

$= \$3108$

b. $I = Prt$

$= \$10\,100 \times 0.1275 \times 4.25$

$= \$5472.9375$

$= \$5472.94$

c. $I = Prt$

$= \$107\,432 \times 0.085 \times 3.5$

$= \$31\,961.02$

2. a. $A = P(1+i)^n$

$$\begin{aligned} &= \$5600(1+0.0925)^6 \\ &= \$5600(1.0925)^6 \\ &= \$5600(1.700\,312\,211) \\ &= \$9521.748\,383 \\ &= \$9521.75 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$9521.75 - \$5600.00 \\ &= \$3921.75 \end{aligned}$$

b. $A = P(1+i)^n$

$$\begin{aligned} &= \$10\,100(1+0.031\,875)^{16} \\ &= \$10\,100(1.031\,875)^{16} \\ &= \$10\,100 \times 1.652\,089\,038 \\ &= \$16\,686.099\,29 \\ &= \$16\,686.10 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$16\,686.10 - \$10\,100.00 \\ &= \$6586.10 \end{aligned}$$

c. $A = P(1+i)^n$

$$\begin{aligned} &= \$107\,432\left(1+\frac{0.085}{12}\right)^{42} \\ &= \$107\,432(1.007\,083\,333)^{42} \\ &= \$107\,432 \times 1.345\,077\,056 \\ &= \$144\,504.318\,3 \\ &= \$144\,504.32 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$144\,504.32 - \$107\,432.00 \\ &= \$37\,072.32 \end{aligned}$$

Extensions

1. a. $I = Prt$

$$\begin{aligned} &= \$1050 \times 0.075 \times 4 \\ &= \$315 \end{aligned}$$

b. $A = P(1+ni)$

$$\begin{aligned} &= \$1050(1+8\times 0.0375) \\ &= \$1050(1+0.3) \\ &= \$1050 \times 1.3 \\ &= \$1365 \end{aligned}$$

$I = A - P$

$$\begin{aligned} &= \$1365 - \$1050 \\ &= \$315 \end{aligned}$$

2. $$\begin{aligned} PV &= A(1+i)^{-n} \\ &= \$1225(1+0.0475)^{-8} \\ &= \$1225(1.0475)^{-8} \\ &= \frac{\$1225}{(1.0475)^{8}} \\ &= \frac{\$1225}{1.449\,546\,839} \\ &= \$845.091\,698\,6 \\ &= \$845.09 \end{aligned}$$

This investment requires \$845.09 now to result in \$1225 in four years if the existing conditions are equal.

3. $$\begin{aligned} PV &= A(1+i)^{-n} \\ &= \$5350(1+0.009\,791\,667)^{-60} \\ &= \$5350(1.009\,791\,667)^{-60} \\ &= \frac{\$5350}{(1.009\,791\,667)^{60}} \\ &= \frac{\$5350}{1.794\,349\,091} \\ &= \$2981.582\,584 \\ &= \$2981.58 \end{aligned}$$

This investment requires \$2981.58 now to result in \$5350 in five years if the existing conditions are equal.

Exploring Topic 2

Activity 1

Define annuity.

1. An annuity is a sequence of equal payments made at regular intervals over a chosen period of time.
2. Simple interest is the amount of interest due at the end of a certain period.
3. Compound interest is interest computed upon the principal for the first period, upon the principal and the first period's interest for the second period, upon the new principal and the second period's interest for the third period, and so on until the time period has expired.
4. The term of an annuity is the time from the beginning of the first payment interval to the end of the last one.
5. The payment interval of an annuity is the time between successive payment dates.
6. The periodic payment for an annuity is the amount paid at the beginning of each of the payment intervals.

Activity 2

Develop a repeating process to find the amount and the present value of an annuity.

1. a. The term of the annuity is ten years.

   b. The annual interest rate is 11%/a.

   c. The interest is compounded once every year.

   d. The payment interval is one year and the payment is $450.

   e.
$$\begin{aligned}A &= \$450(1.11)^9 + \$450(1.11)^8 + \$450(1.11)^7\\ &\quad + \$450(1.11)^6 + \$450(1.11)^5 + \$450(1.11)^4\\ &\quad + \$450(1.11)^3 + \$450(1.11)^2 + \$450(1.11) + \$450\\ &= \$1151.12 + \$1037.04 + \$934.27 + \$841.69 + \$758.28\\ &\quad + \$683.13 + \$615.43 + \$554.45 + \$499.50 + \$450.00\\ &= \$7524.91\end{aligned}$$

   The amount of the annuity is $7524.91.

2. a. The term of the annuity is six years.

   b. The annual interest is 15%/a.

   c. The interest is compounded every six months.

   d. The payment interval is every half year and the payment is $700.

   e.
$$\begin{aligned}A &= \$700(1.075)^{11} + \$700(1.075)^{10} + \$700(1.075)^9\\ &\quad + \$700(1.075)^8 + \$700(1.075)^7 + \$700(1.075)^6\\ &\quad + \$700(1.075)^5 + \$700(1.075)^4 + \$700(1.075)^3\\ &\quad + \$700(1.075)^2 + \$700(1.075) + \$700\\ &= \$1550.93 + \$1442.72 + \$1342.07 + \$1248.44\\ &\quad + \$1161.33 + \$1080.31 + \$1004.94 + \$934.83\\ &\quad + \$869.61 + \$808.94 + \$752.50 + \$700.00\\ &= \$12\ 896.62\end{aligned}$$

   The amount of the annuity is $12 896.62.

3.
$$\begin{aligned}PV &= \frac{\$2500}{1.0925} + \frac{\$2500}{(1.0925)^2} + \frac{\$2500}{(1.0925)^3} + \frac{\$2500}{(1.0925)^4} + \frac{\$2500}{(1.0925)^5}\\ &= \$2288.33 + \$2094.58 + \$1917.24 + \$1754.91 + \$1606.32\\ &= \$9661.38\end{aligned}$$

   The Kelseys must invest $9661.38 now to make this annuity possible.

4. $$\begin{aligned} PV &= \frac{\$950}{1.07}+\frac{\$950}{(1.07)^2}+\frac{\$950}{(1.07)^3}+\frac{\$950}{(1.07)^4}+\frac{\$950}{(1.07)^5}+\frac{\$950}{(1.07)^6} \\ &\quad +\frac{\$950}{(1.07)^7}+\frac{\$950}{(1.07)^8}+\frac{\$950}{(1.07)^9}+\frac{\$950}{(1.07)^{10}}+\frac{\$950}{(1.07)^{11}} \\ &\quad +\frac{\$950}{(1.07)^{12}}+\frac{\$950}{(1.07)^{13}}+\frac{\$950}{(1.07)^{14}}+\frac{\$950}{(1.07)^{15}}+\frac{\$950}{(1.07)^{16}} \\ &= \$887.85+\$829.77+\$775.48+\$724.75+\$677.34 \\ &\quad +\$633.03+\$591.61+\$552.91+\$516.74+\$482.93 \\ &\quad +\$451.34+\$421.81+\$394.22+\$368.43+\$344.32 \\ &\quad +\$321.80 \\ &= \$8974.33 \end{aligned}$$

The present value of the annuity is \$8974.33.

Extra Help

1. a. The term of the annuity is three years.

b. The annual interest rate is 12%/a.

c. Interest is compounded every three months or quarterly.

d. The payment interval is every three months and the payment is \$150.

e. $$\begin{aligned} A &= \$150(1.03)^{11}+\$150(1.03)^{10}+\$150(1.03)^{9} \\ &\quad +\$150(1.03)^{8}+\$150(1.03)^{7}+\$150(1.03)^{6} \\ &\quad +\$150(1.03)^{5}+\$150(1.03)^{4}+\$150(1.03)^{3} \\ &\quad +\$150(1.03)^{2}+\$150(1.03)+\$150 \\ &= \$207.64+\$201.59+\$195.72+\$190.02 \\ &\quad +\$184.48+\$179.11+\$173.89+\$168.83 \\ &\quad +\$163.91+\$159.14+\$154.50+\$150.00 \\ &= \$2128.83 \end{aligned}$$

The amount of the annuity is \$2128.83.

2. a. The term of the annuity is three years.

b. The annual interest rate is 9%/a.

c. The interest is compounded every four months or every third of a year.

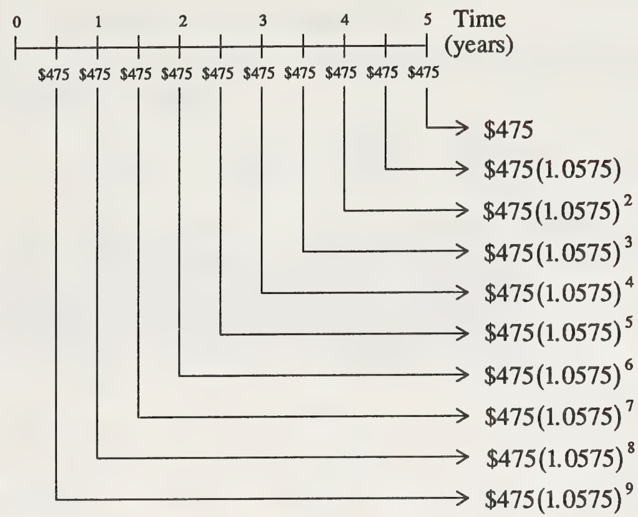
d. The payment interval is every four months or every third of a year and the payment is \$250.

e. $$\begin{aligned} PV &= \frac{\$250}{1.03}+\frac{\$250}{(1.03)^2}+\frac{\$250}{(1.03)^3}+\frac{\$250}{(1.03)^4}+\frac{\$250}{(1.03)^5} \\ &\quad +\frac{\$250}{(1.03)^6}+\frac{\$250}{(1.03)^7}+\frac{\$250}{(1.03)^8}+\frac{\$250}{(1.03)^9} \\ &= \$242.72+\$235.65+\$228.79+\$222.12+\$215.65 \\ &\quad +\$209.37+\$203.27+\$197.35+\$191.60 \\ &= \$1946.52 \end{aligned}$$

The present value of the annuity is \$1946.52.

Extensions

1. Annuity Time Line (Amount)



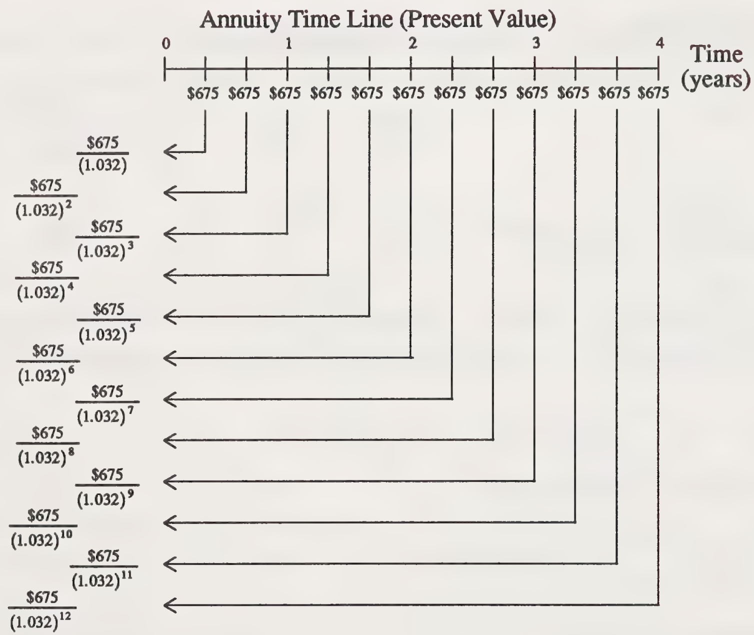
$$A = \$475(1.0575)^9 + \$475(1.0575)^8 + \$475(1.0575)^7 + \$475(1.0575)^6 + \$475(1.0575)^5$$
$$+ \$475(1.0575)^4 + \$475(1.0575)^3 + \$475(1.0575)^2 + \$475(1.0575) + \$475$$

$$= \$785.63 + \$742.91 + \$702.52 + \$664.32 + \$628.20 + \$594.04 + \$561.74 + \$531.20 + \$502.31 + \$475.00$$

$$= \$6187.87$$

The amount is the value of the investment at the end of five years. In this case the value is $6187.87.

2.



$$PV = \frac{\$675}{(1.032)} + \frac{\$675}{(1.032)^2} + \frac{\$675}{(1.032)^3} + \frac{\$675}{(1.032)^4} + \frac{\$675}{(1.032)^5} + \frac{\$675}{(1.032)^6} + \frac{\$675}{(1.032)^7} + \frac{\$675}{(1.032)^8}$$

$$+ \frac{\$675}{(1.032)^9} + \frac{\$675}{(1.032)^{10}} + \frac{\$675}{(1.032)^{11}} + \frac{\$675}{(1.032)^{12}}$$

$$= \$654.07 + \$633.79 + \$614.14 + \$595.09 + \$576.64 + \$558.76 + \$541.43 + \$524.65 + \$508.38$$

$$+ \$492.61 + \$477.34 + \$462.54$$

$$= \$6639.44$$

The present value of $6639.44 can mean that this much must be invested now if payments of $675 are to be made at four-month intervals for four years at 9.6%/a compounded once every four months.

Exploring Topic 3

Activity 1

Determine the amount of an annuity by using tables.

1. $A = Rs_{\overline{n}|i}$
$= \$325 \times 61.222\,61$
$= \$19\,897.35$

$R = \$325$
$n = 4 \times 12$
$= 48$
$i = 12\% \div 12$
$= 1\%$

2. $A = Rs_{\overline{n}|i}$
$= \$1011 \times 29.778\,08$
$= \$30\,105.64$

$R = \$1011$
$n = 10 \times 2$
$= 20$
$i = 8\% \div 2$
$= 4\%$

3. $A = Rs_{\overline{n}|i}$
$= \$3064.15 \times 25.544\,66$
$= \$78\,272.67$

$R = \$3064.15$
$n = 5 \times 4$
$= 20$
$i = 10\% \times \frac{1}{4}$
$= 2\frac{1}{2}\%$

4. $A = Rs_{\overline{n}|i}$
$= \$75.49 \times 35.719\,25$
$= \$2696.45$

$R = \$75.49$
$n = 7 \times 3$
$= 21$
$i = 15\% \div 3$
$= 5\%$

5. $A = Rs_{\overline{n}|i}$
$= \$2004.16 \times 69.565\,22$
$= \$139\,419.83$

$R = \$2004.16$
$n = 4 \times 12$
$= 48$
$i = 18\% \div 12$
$= 1\frac{1}{2}\%$

Activity 2

Determine the amount of an annuity by applying the formula $A = \frac{R\left[(1+i)^n - 1\right]}{i}$.

1. Formula Method

$R = \$350.22$
$i = 6\% \div 2$
$= 3\%$
$n = 5 \times 2$
$= 10$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$350.22\left[(1+0.03)^{10} - 1\right]}{0.03}$$

$$= \frac{\$120.4464}{0.03}$$

$= \$4014.88$ (to the nearest cent)

Table Method

$A = Rs_{\overline{n|}i}$
$= \$350.22 \times 11.463\,88$
$= \$4014.880\,054$
$= \$4014.88$ (to the nearest cent)

2. Formula Method

$R = \$916.14$
$i = 16\% \div 4$
$= 4\%$
$n = 7 \times 4$
$= 28$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$916.14\left[(1+0.04)^{28} - 1\right]}{0.04}$$

$$= \frac{\$1831.0921}{0.04}$$

$= \$45\,777.30$ (to the nearest cent)

Table Method

$A = Rs_{\overline{n|}i}$
$= \$916.14 \times 49.967\,58$
$= \$45\,777.298\,74$
$= \$45\,777.30$ (to the nearest cent)

3. Formula Method

$$R = \$1160.15 \qquad i = 9\% \div 1 = 9\% \qquad n = 20 \times 1 = 20$$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$1160.15\left[(1+0.09)^{20} - 1\right]}{0.09}$$

$$= \frac{\$5341.8072}{0.09}$$

$$= \$59\,353.41 \text{ (to the nearest cent)}$$

Table Method

$$A = Rs_{\overline{n}|i}$$
$$= \$1160.15 \times 51.160\,12$$
$$= \$59\,353.413\,22$$
$$= \$59\,353.41 \text{ (to the nearest cent)}$$

4. Formula Method

$$R = \$955.60 \qquad i = 18\% \div 12 = 1\tfrac{1}{2}\% \qquad n = 3 \times 12 = 36$$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$955.60\left[(1+0.015)^{36} - 1\right]}{0.015}$$

$$= \frac{\$677.653\,74}{0.015}$$

$$= \$45\,176.92 \text{ (to the nearest cent)}$$

Table Method

$$A = Rs_{\overline{n}|i}$$
$$= \$955.60 \times 47.275\,97$$
$$= \$45\,176.916\,93$$
$$= \$45\,176.92 \text{ (to the nearest cent)}$$

5. Formula Method

$R = \$2007.10$
$i = 15\% \div 3$
$= 5\%$
$n = 10 \times 3$
$= 30$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$2007.10\left[(1+0.05)^{30} - 1\right]}{0.05}$$

$$= \frac{\$6667.4705}{0.05}$$

$= \$133\,349.41$ (to the nearest cent)

Table Method

$A = Rs_{\overline{n}|i}$
$= \$2007.10 \times 66.438\,85$
$= \$133\,349.4158$
$= \$133\,349.42$ (to the nearest cent)

This answer is close to the answer obtained by the formula method.

Activity 3

Determine the present value of an annuity by using tables.

1. $PV = Ra_{\overline{n}|i}$
$= \$350.62 \times 8.530\,20$
$= \$2990.858\,724$
$= \$2990.86$ (to the nearest cent)

$R = \$350.62$
$n = 5 \times 2$
$= 10$
$i = 6\% \div 2$
$= 3\%$

2. $PV = Ra_{\overline{n}|i}$
$= \$916.14 \times 16.663\,06$
$= \$15\,265.695\,79$
$= \$15\,265.70$ (to the nearest cent)

$R = \$916.14$
$n = 7 \times 4$
$= 28$
$i = 16\% \div 4$
$= 4\%$

3. $PV = Ra_{\overline{n}|i}$
$= \$1160.15 \times 9.128\,55$
$= \$10\,590.487\,28$
$= \$10\,590.49$ (to the nearest cent)

$R = \$1160.15$
$n = 20 \times 1$
$= 20$
$i = 9\% \div 1$
$= 9\%$

4. $PV = Ra_{\overline{n}|i}$

$$\begin{aligned} &= \$955.60 \times 27.660\,68 \\ &= \$26\,432.545\,81 \\ &= \$26\,432.55 \text{ (to the nearest cent)} \end{aligned}$$

$$\begin{aligned} R &= \$955.60 \\ n &= 3 \times 12 \\ &= 36 \\ i &= 18\% \div 12 \\ &= 1\tfrac{1}{2}\% \end{aligned}$$

5. $PV = Ra_{\overline{n}|i}$

$$\begin{aligned} &= \$2007.10 \times 15.372\,45 \\ &= \$30\,854.044\,4 \\ &= \$30\,854.04 \text{ (to the nearest cent)} \end{aligned}$$

$$\begin{aligned} R &= \$2007.10 \\ n &= 10 \times 3 \\ &= 30 \\ i &= 15\% \div 3 \\ &= 5\% \end{aligned}$$

Activity 4

Determine the present value of an annuity by applying the formula $PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$.

1. $PV = \dfrac{R\left[1-(1+i)^{-n}\right]}{i}$

$$\begin{aligned} A &= \$350.62 \\ i &= 6\% \div 2 \\ &= 3\% \\ n &= 5 \times 2 \\ &= 10 \end{aligned}$$

$$\begin{aligned} &= \frac{\$350.62\left[1-(1+0.03)^{-10}\right]}{0.03} \\ &= \frac{\$350.62\left[1-(1.03)^{-10}\right]}{0.03} \\ &= \frac{\$89.725\,792}{0.03} \\ &= \$2990.859\,718 \\ &= \$2990.86 \text{ (to the nearest cent)} \end{aligned}$$

2. $PV = \dfrac{R\left[1-(1+i)^{-n}\right]}{i}$

$$\begin{aligned} A &= \$916.14 \\ i &= 16\% \div 4 \\ &= 4\% \\ n &= 7 \times 4 \\ &= 28 \end{aligned}$$

$$\begin{aligned} &= \frac{\$916.14\left[1-(1+0.04)^{-28}\right]}{0.04} \\ &= \frac{\$916.14\left[1-(1.04)^{-28}\right]}{0.04} \\ &= \frac{\$610.627\,95}{0.04} \\ &= \$15\,265.698\,75 \\ &= \$15\,265.70 \text{ (to the nearest cent)} \end{aligned}$$

3. $PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$

$A = \$1160.15$

$i = 9\% \div 1$

$= 9\%$

$n = 20 \times 1$

$= 20$

$= \frac{\$1160.15\left[1-(1+0.09)^{-20}\right]}{0.09}$

$= \frac{\$1160.15\left[1-(1.09)^{-20}\right]}{0.09}$

$= \frac{\$953.143\,403\,2}{0.09}$

$= \$10\,590.482\,26$

$= \$10\,590.48$ (to the nearest cent)

4. $PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$

$A = \$955.60$

$i = 18\% \div 12$

$= 1\frac{1}{2}\%$

$n = 3 \times 12$

$= 36$

$= \frac{\$955.60\left[1-(1+0.015)^{-36}\right]}{0.015}$

$= \frac{\$955.60\left[1-(1.015)^{-36}\right]}{0.015}$

$= \frac{\$396.488\,248\,9}{0.015}$

$= \$26\,432.549\,93$

$= \$26\,432.55$ (to the nearest cent)

5. $PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$

$A = \$2007.10$

$i = 15\% \div 3$

$= 5\%$

$n = 10 \times 3$

$= 30$

$= \frac{\$2007.10\left[1-(1+0.05)^{-30}\right]}{0.05}$

$= \frac{\$2007.10\left[1-(1.05)^{-30}\right]}{0.05}$

$= \frac{\$1542.702\,323}{0.05}$

$= \$30\,854.046\,45$

$= \$30\,854.05$ (to the nearest cent)

Activity 5

Solve word problems involving the amount and the present value of an annuity.

1. Find the amount of the annuity.

Table Method

$$\begin{aligned} A &= Rs_{\overline{n}|i} \\ &= \$750 \times 12.577\,89 \\ &= \$9433.4175 \\ &= \$9433.42 \end{aligned}$$

$$\begin{aligned} R &= \$750 \\ n &= 5 \times 2 \\ &= 10 \\ i &= 10\% \div 2 \\ &= 5\% \end{aligned}$$

The cash available at the end of five years would amount to $9433.42.

Formula Method

$$\begin{aligned} A &= \frac{R\left[(1+i)^n - 1\right]}{i} \\ &= \frac{\$750\left[(1+0.05)^{10} - 1\right]}{0.05} \\ &= \frac{\$750\left[(1.05)^{10} - 1\right]}{0.05} \\ &= \frac{\$471.670\,97}{0.05} \\ &= \$9433.4194 \\ &= \$9433.42 \end{aligned}$$

$$\begin{aligned} R &= \$750 \\ i &= 10\% \div 2 \\ &= 5\% \\ n &= 5 \times 2 \\ &= 10 \end{aligned}$$

The amount available at the end of five years would be $9433.42.

2. Find the amount of the annuity.

Table Method

$$\begin{aligned} A &= Rs_{\overline{n}|i} \\ &= \$525 \times 32.349\,04 \\ &= \$16\,983.246 \\ &= \$16\,983.25 \\ &= \$16\,983 \text{ (to the nearest dollar)} \end{aligned}$$

$$\begin{aligned} R &= \$525 \\ n &= 6 \times 4 \\ &= 24 \\ i &= 10\% \div 4 \\ &= 2\tfrac{1}{2}\% \end{aligned}$$

The sum available at the end of the sixth year is $16 983.

Formula Method

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$525\left[(1+0.025)^{24} - 1\right]}{0.025}$$

$$= \frac{\$525\left[(1.025)^{24} - 1\right]}{0.025}$$

$$= \frac{\$424.581\,123\,5}{0.025}$$

$$= \$16\,983.244\,94$$

$$= \$16\,983.24$$

$$= \$16\,983 \text{ (to the nearest dollar)}$$

$R = \$525$

$i = 10\% \div 4$

$= 2\frac{1}{2}\%$

$n = 6 \times 4$

$= 24$

The sum available at the end of the sixth year is $16 983.

3. Find the present value of the annuity.

Table Method

$$PV = Ra_{\overline{n}|i}$$

$$= \$450 \times 34.042\,55$$

$$= \$15\,319.1475$$

$$= \$15\,319.15$$

$R = \$450$

$n = 4 \times 12$

$= 48$

$i = 18\% \div 12$

$= 1\frac{1}{2}\%$

The cash price of the car would be $15 319.15.

Formula Method

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

$$= \frac{\$450\left[1-(1+0.015)^{-48}\right]}{i}$$

$$= \frac{\$450\left[1-(1.015)^{-48}\right]}{0.015}$$

$$= \frac{\$229.787\,237\,1}{0.015}$$

$$= \$15\,319.149\,14$$

$$= \$15\,319.15$$

$R = \$450$

$i = 18\% \div 12$

$= 1\frac{1}{2}\%$

$n = 4 \times 12$

$= 48$

The cash price of the car would be $15 319.15.

4. Find the present value of the annuity.

Table Method

$$\begin{aligned} PV &= Ra_{\overline{n|}i} \\ &= \$750 \times 13.590\,33 \\ &= \$10\,192.7475 \\ &= \$10\,192.75 \end{aligned} \qquad \begin{aligned} R &= \$750 \\ n &= 10 \times 2 \\ &= 20 \\ i &= 8\% \div 2 \\ &= 4\% \end{aligned}$$

The amount which must be invested now is \$10 192.75.

Formula Method

$$\begin{aligned} PV &= \frac{R\left[1-(1+i)^{-n}\right]}{i} \\ &= \frac{\$750\left[1-(1+0.04)^{-20}\right]}{0.04} \\ &= \frac{\$407.709\,979\,03}{0.04} \\ &= \$10\,192.744\,76 \\ &= \$10\,192.74 \end{aligned} \qquad \begin{aligned} R &= \$750 \\ i &= 8\% \div 2 \\ &= 4\% \\ n &= 10 \times 2 \\ &= 20 \end{aligned}$$

The amount which must be invested now is \$10 192.74.

5. Find the present value of the annuity.

Table Method

$$\begin{aligned} PV &= Ra_{\overline{n|}i} \\ &= \$1500 \times 13.753\,51 \\ &= \$20\,630.265 \\ &= \$20\,630.27 \end{aligned} \qquad \begin{aligned} R &= \$1500 \\ n &= 6 \times 3 \\ &= 18 \\ i &= 9\% \div 3 \\ &= 3\% \end{aligned}$$

The amount in the annuity at the present time is \$20 630.27.

Formula Method

$$\begin{aligned} PV &= \frac{R\left[1-(1+i)^{-n}\right]}{i} \\ &= \frac{\$1500\left[1-(1+0.03)^{-18}\right]}{0.03} \\ &= \frac{\$618.908\,088\,6}{0.03} \\ &= \$20\,630.269\,62 \\ &= \$20\,630.27 \end{aligned} \qquad \begin{aligned} R &= \$1500 \\ i &= 9\% \div 3 \\ &= 3\% \\ n &= 6 \times 3 \\ &= 18 \end{aligned}$$

The amount in the annuity at the present time is \$20 630.27.

6. Find the amount of the annuity.

Table Method

$$A = Rs_{\overline{n}|i}$$
$$= \$85 \times 61.222\,61$$
$$= \$5203.921\,85$$
$$= \$5203.92$$

$R = \$85$

$n = 4 \times 12$
$= 48$

$i = 12\% \div 12$
$= 1\%$

The twins will have $5203.92 in their account at the end of four years.

Formula Method

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$
$$= \frac{\$85\left[(1+0.01)^{48} - 1\right]}{0.01}$$
$$= \frac{\$52.039\,216\,6}{0.01}$$
$$= \$5203.921\,66$$
$$= \$5203.92$$

$R = \$85$

$i = 12\% \div 12$
$= 1\%$

$n = 4 \times 12$
$= 48$

The twins will have $5203.92 in their account at the end of four years.

7. Find the present value of the annuity.

Table Method

$$PV = Ra_{\overline{n}|i}$$
$$= \$149 \times 32.552\,34$$
$$= \$4850.298\,66$$
$$= \$4850.30$$

$R = \$149$

$n = 3\frac{3}{4} \times 12$
$= 45$

$i = 18\% \div 12$
$= 1\frac{1}{2}\%$

The cash price of the motorbike would be $4850.30.

Formula Method

$$PV = \frac{R\left[1 - (1+i)^{-n}\right]}{i}$$
$$= \frac{\$149\left[1 - (1+0.015)^{-45}\right]}{0.015}$$
$$= \frac{\$72.754\,473\,59}{0.015}$$
$$= \$4850.298\,239$$
$$= \$4850.30$$

$R = \$149$

$i = 18\% \div 12$
$= 1\frac{1}{2}\%$

$n = 3\frac{3}{4} \times 12$
$= 45$

The cash price of the motorbike would be $4850.30.

By paying \$149 every month for $3\frac{3}{4}$ years, the total cost would be $\$149 \times 45 = \$6705$.

By paying cash, they would save $\$6705.00 - \$4850.30 = \$1854.70$.

Extra Help

1. a. $A = Rs_{\overline{n}|i}$ $\quad R = \$550$

$$\begin{aligned} A &= \$550 \times 39.859\,80 & n &= 7 \times 4 \\ &= \$21\,922.89 & &= 28 \\ & & i &= 10\% \div 4 \\ & & &= 2\tfrac{1}{2}\% \end{aligned}$$

This annuity amounts to \$21 922.89 in seven years.

b. $A = Rs_{\overline{n}|i}$ $\quad R = \$1034.16$

$$\begin{aligned} A &= \$1034.16 \times 58.176\,67 & n &= 12 \times 2 \\ &= \$60\,163.985\,05 & &= 24 \\ &= \$60\,163.99 & i &= 14\% \div 2 \\ & & &= 7\% \end{aligned}$$

This annuity amounts to \$60 163.99 in twelve years.

c. $A = Rs_{\overline{n}|i}$ $\quad R = \$139.43$

$$\begin{aligned} A &= \$139.43 \times 69.565\,22 & n &= 4 \times 12 \\ &= \$9699.478\,625 & &= 48 \\ &= \$9699.48 & i &= 18\% \div 12 \\ & & &= 1\tfrac{1}{2}\% \end{aligned}$$

This annuity amounts to \$9699.48 in four years.

2. a. $PV = Ra_{\overline{n}|i}$ $\quad R = \$75$

$$\begin{aligned} PV &= \$75 \times 10.379\,66 & n &= 5 \times 3 \\ &= \$778.4745 & &= 15 \\ &= \$778.47 & i &= 15\% \div 3 \\ & & &= 5\% \end{aligned}$$

The present value of this annuity is \$778.47.

b. $PV = Ra_{\overline{n}|i}$ $\quad R = \$2003.15$

$$\begin{aligned} PV &= \$2003.15 \times 4.766\,54 & n &= 6 \times 1 \\ &= \$9548.094\,601 & &= 6 \\ &= \$9548.09 & i &= 7\% \div 1 \\ & & &= 7\% \end{aligned}$$

The present value of this annuity is \$9548.09.

c. $PV = Ra_{\overline{n}|i}$

$$= \$511 \times 24.015\,84$$
$$= \$12\,272.094\,24$$
$$= \$12\,272.09$$

$R = \$511$

$n = 2\frac{1}{2} \times 12$

$= 30$

$i = 18\% \div 12$

$= 1\frac{1}{2}\%$

The present value of this annuity is $12 272.09.

Extensions

1.

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$
$$\$2200 = \frac{R\left[(1+0.03)^{10} - 1\right]}{0.03}$$
$$\$2200 = \frac{R\left[(1.03)^{10} - 1\right]}{0.03}$$
$$\$2200 = \frac{R(0.343\,916\,379)}{0.03}$$
$$R(0.343\,916\,379) = \$2200 \times 0.03$$
$$R(0.343\,916\,379) = \$66$$
$$R = \$66 \div 0.343\,916\,379$$
$$= \$191.907\,114\,5$$
$$= \$191.91$$

$A = \$2200$

$n = 2\frac{1}{2} \times 4$

$= 10$

$i = 12\% \div 4$

$= 3\%$

$= 0.03$

$R = ?$

Check the answer as follows:

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$
$$= \frac{\$191.91\left[(1+0.03)^{10} - 1\right]}{0.03}$$
$$= \frac{\$191.91(0.343\,916\,379)}{0.03}$$
$$= \frac{\$66.000\,992\,35}{0.03}$$
$$= \$2200.033\,078$$
$$= \$2200.03$$

This is as close as you will get to $2200.

Monica must deposit $191.91 every quarter year for $2\frac{1}{2}$ years to save $2200 to buy the stereo set.

2. Plan A is as follows:

$$\begin{aligned} A &= \frac{R\left[(1+i)^n - 1\right]}{i} \\ &= \frac{\$600\left[(1+0.055)^8 - 1\right]}{0.055} \\ &= \frac{\$320.811\,908\,9}{0.055} \\ &= \$5832.943\,799 \\ &= \$5832.94 \end{aligned}$$

Rupert would have $5832.94 if he chooses Plan A.

Plan B is as follows:

$$\begin{aligned} A &= \frac{R\left[(1+i)^n - 1\right]}{i} \\ &= \frac{\$300\left[(1+0.025)^{16} - 1\right]}{0.025} \\ &= \frac{\$145.351\,686\,2}{0.025} \\ &= \$5814.067\,447 \\ &= \$5814.07 \end{aligned}$$

Rupert would have $5814.07 if he chooses Plan B.

The difference in the amounts is as follows:

$$\begin{aligned} D &= \text{Plan A} - \text{Plan B} \\ &= \$5832.94 - \$5814.07 \\ &= \$18.87 \end{aligned}$$

The difference in the amounts is $18.87 if he chooses Plan A over Plan B.

3. Igora's investment is as follows:

$$\begin{aligned} A &= \frac{R\left[(1+i)^n - 1\right]}{i} \\ &= \frac{\$75\left[(1+0.04)^{15} - 1\right]}{0.04} \\ &= \frac{\$60.070\,762\,9}{0.04} \\ &= \$1501.769\,073 \\ &= \$1501.77 \end{aligned}$$

Igora has $1501.77 under the conditions outlined.

Redeana's investment is as follows:

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$45\left[(1+0.0075)^{60} - 1\right]}{0.0075}$$

$$= \frac{\$25.455\,646\,21}{0.0075}$$

$$= \$3394.086\,161$$

$$= \$3394.09$$

Redeana has $3394.09 under the conditions outlined.

The difference in the two amounts is as follows:

$$\begin{aligned} D &= \text{Redeana} - \text{Igora} \\ &= \$3394.09 - \$1501.77 \\ &= \$1892.32 \end{aligned}$$

The difference in the two amounts is $1892.32.

4. a.

$$\begin{aligned} A &= \$20\,000 \\ i &= 11\tfrac{1}{2}\% \div 1 \\ &= 11\tfrac{1}{2}\% \\ n &= 25 \times 1 \\ &= 25 \\ R &= ? \end{aligned}$$

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$\$20\,000 = \frac{R\left[(1+0.115)^{25} - 1\right]}{0.115}$$

$$\$20\,000 = \frac{R \times 14.200\,983\,39}{0.115}$$

$$14.200\,983\,39R = 0.115 \times \$20\,000$$

$$R = \frac{0.115 \times \$20\,000}{14.200\,983\,39}$$

$$= \$161.960\,614\,8$$

$$= \$161.96$$

Check the answer as follows:

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$A = \frac{\$161.96\left[(1+0.115)^{25} - 1\right]}{0.115}$$

$$A = \frac{\$2299.991\,27}{0.115}$$

$$A = \$19\,999.924\,09$$

$$A = \$19\,999.92$$

This is as close as you can get to $20 000.

The periodic payment would be about $161.96.
If you used $161.97 for R, then A would equal $20 001.16.
Therefore, the periodic payment is $161.96.

b.

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$\$20\,000 = \frac{R\left[(1+0.0575)^{50} - 1\right]}{0.0575}$$

$$\$20\,000 = \frac{R \times 15.368\,873\,87}{0.0575}$$

$$15.368\,873\,87R = 0.0575 \times \$20\,000$$

$$R = \frac{0.0575 \times \$20\,000}{15.368\,873\,87}$$

$$= \$74.826\,562\,42$$

$$= \$74.83$$

$A = \$20\,000$

$i = 11\frac{1}{2}\% \div 2$

$= 5\frac{3}{4}\%$

$= 0.0575$

$n = 25 \times 2$

$= 50$

$R = ?$

Check the answer as follows:

$$A = \frac{R\left[(1+i)^n - 1\right]}{i}$$

$$= \frac{\$74.83\left[(1+0.0575)^{50} - 1\right]}{0.0575}$$

$$= \frac{\$1150.052\,832}{0.0575}$$

$$= \$20\,000.918\,82$$

$$= \$20\,000.92$$

This is as close as you can get to $20 000.

The periodic payment would be about $74.83 since $74.82 would give you $19 998.25 and $74.84 would result in $20 003.59.

Appendix B
Tables

Table 1 Amount of an Annuity

$$A = Rs_{\overline{n}|i}$$

n	$\frac{1}{2}\%$	1%	$1\frac{1}{2}\%$	2%	$2\frac{1}{2}\%$	3%	$3\frac{1}{2}\%$	n
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1
2	2.00500	2.01000	2.01500	2.02000	2.02500	2.03000	2.03500	2
3	3.01503	3.03010	3.04523	3.06040	3.07563	3.09090	3.10623	3
4	4.03010	4.06040	4.09090	4.12161	4.15252	4.18363	4.21404	4
5	5.05025	5.10101	5.15227	5.20404	5.25633	5.30914	5.36247	5
6	6.07550	6.15202	6.22955	6.30812	6.38774	6.46841	6.55015	6
7	7.10588	7.21354	7.32299	7.43428	7.54743	7.66246	7.77948	7
8	8.14141	8.28567	8.43284	8.58297	8.73612	8.89237	9.05169	8
9	9.18212	9.36853	9.55933	9.75463	9.95452	10.15911	10.36850	9
10	10.22803	10.46221	10.70272	10.94972	11.20338	11.46388	11.73139	10
11	11.27917	11.56683	11.86326	12.16872	12.48347	12.80780	13.14199	11
12	12.33556	12.68250	13.04121	13.41209	13.79555	14.19203	14.60196	12
13	13.39724	13.80933	14.23683	14.68033	15.14044	15.61779	16.11303	13
14	14.46423	14.94742	15.45088	15.97394	16.51895	17.08632	17.67698	14
15	15.53655	16.09690	16.68214	17.29342	17.93193	18.59891	19.29568	15
16	16.61423	17.25786	17.93237	18.63929	19.38022	20.15688	20.97103	16
17	17.69730	18.43044	19.20136	20.01207	20.86473	21.76159	22.70502	17
18	18.78579	19.61475	20.48938	21.41231	22.38635	23.41444	24.49969	18
19	19.87972	20.81090	21.79672	22.84056	23.94601	25.11687	26.35718	19
20	20.97912	22.01900	23.12367	24.29737	25.54466	26.87037	28.27968	20
21	22.08401	23.23919	24.47052	25.78332	27.18327	28.67649	30.26947	21
22	23.19443	24.47159	25.83758	27.29898	28.86286	30.53678	32.32890	22
23	24.31040	25.71630	27.22514	28.84496	30.58443	32.45288	34.46041	23
24	25.43196	26.97346	28.63352	30.42186	32.34904	34.42647	36.66653	24
25	26.55912	28.24320	30.06302	32.03030	34.15776	36.45926	38.94986	25
26	27.69191	29.52563	31.51397	33.67091	36.01171	38.55304	41.31310	26
27	28.83037	30.82089	32.98668	35.34432	37.91200	40.70963	43.75906	27
28	29.97452	32.12910	34.48148	37.05121	39.85980	42.93092	46.29063	28
29	31.12439	33.45039	35.99870	38.79223	41.85630	45.21885	48.91080	29
30	32.28002	34.78489	37.53868	40.56808	43.90270	47.57546	51.62268	30
31	33.44142	36.13274	39.10176	42.37944	46.00027	50.00268	54.42947	31
32	34.60862	37.49408	40.68829	44.22703	48.15028	52.50276	57.33450	32
33	35.78167	38.86901	42.29861	46.11157	50.35403	55.07784	60.34121	33
34	36.96058	40.25770	43.93309	48.03380	52.61289	57.73018	63.45315	34
35	38.14538	41.66028	45.59209	49.99448	54.92821	60.46208	66.67401	35
36	39.33611	43.07688	47.27597	51.99437	57.30141	63.27594	70.00760	36
37	40.53279	44.50765	48.98511	54.03425	59.73395	66.17422	73.45787	37
38	41.73545	45.95272	50.71989	56.11494	62.22730	69.15945	77.02889	38
39	42.94413	47.41225	52.48068	58.23724	64.78298	72.23423	80.72491	39
40	44.15885	48.88637	54.26789	60.40198	67.40255	75.40126	84.55028	40
41	45.37964	50.37524	56.08191	62.61002	70.08762	78.66330	88.50954	41
42	46.60654	51.87899	57.92314	64.86222	72.83981	82.02320	92.60787	42
43	47.83957	53.39778	59.79199	67.15947	75.66080	85.48389	96.84863	43
44	49.07877	54.93176	61.68887	69.50266	78.55232	89.04841	101.23831	44
45	50.32416	56.48107	63.61420	71.89271	81.51613	92.71986	105.78162	45
46	51.57579	58.04589	65.56841	74.33056	84.55403	96.50146	110.48403	46
47	52.83366	59.62634	67.55194	76.81718	87.66789	100.39650	115.35097	47
48	54.09783	61.22261	69.56522	79.35352	90.85958	104.40840	120.38826	48
49	55.36832	62.83483	71.60870	81.94059	94.13107	108.54065	125.60185	49
50	56.64516	64.46318	73.68283	84.57940	97.48435	112.79687	130.99791	50
n	$\frac{1}{2}\%$	1%	$1\frac{1}{2}\%$	2%	$2\frac{1}{2}\%$	3%	$3\frac{1}{2}\%$	n

n	4%	5%	6%	7%	8%	9%	10%	n
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1
2	2.04000	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2
3	3.12160	3.15250	3.18360	3.21490	3.24640	3.27810	3.31000	3
4	4.24646	4.31013	4.37462	4.43994	4.50611	4.57313	4.64100	4
5	5.41632	5.52563	5.63709	5.75074	5.86660	5.98471	6.10510	5
6	6.63298	6.80191	6.97532	7.15329	7.33593	7.52333	7.71561	6
7	7.89829	8.14201	8.39384	8.65402	8.92280	9.20043	9.48717	7
8	9.21423	9.54911	9.89747	10.25980	10.63663	11.02847	11.43589	8
9	10.58280	11.02656	11.49132	11.97799	12.48756	13.02104	13.57948	9
10	12.00611	12.57789	13.18079	13.81645	14.48656	15.19293	15.93742	10
11	13.48635	14.20679	14.97164	15.78360	16.64549	17.56029	18.53117	11
12	15.02581	15.91713	16.86994	17.88845	18.97713	20.14072	21.38428	12
13	16.62684	17.71298	18.88214	20.14064	21.49530	22.95338	24.52271	13
14	18.29191	19.59863	21.01507	22.55049	24.21492	26.01919	27.97498	14
15	20.02359	21.57856	23.27597	25.12902	27.15211	29.36092	31.77248	15
16	21.82453	23.65749	25.67253	27.88805	30.32428	33.00330	35.94973	16
17	23.69751	25.84037	28.21288	30.84022	33.75023	36.97370	40.54470	17
18	25.64541	28.13238	30.90565	33.99903	37.45024	41.30134	45.59917	18
19	27.67123	30.53900	33.75999	37.37896	41.44626	46.01846	51.15909	19
20	29.77808	33.06595	36.78559	40.99549	45.76196	51.16012	57.27500	20
21	31.96920	35.71925	39.99273	44.86518	50.42292	56.76453	64.00250	21
22	34.24797	38.50521	43.39229	49.00574	55.45676	62.87334	71.40275	22
23	36.61789	41.43048	46.99583	53.43614	60.89330	69.53194	79.54302	23
24	39.08260	44.50200	50.81558	58.17667	66.76476	76.78981	88.49733	24
25	41.64591	47.72710	54.86451	63.24904	73.10594	84.70090	98.34706	25
26	44.31174	51.11345	59.15638	68.67647	79.95442	93.32398	109.18177	26
27	47.08421	54.66913	63.70577	74.48382	87.35077	102.72313	121.09994	27
28	49.96758	58.40258	68.52811	80.69769	95.33883	112.96822	134.20994	28
29	52.96629	62.32271	73.63980	87.34653	103.96594	124.13536	148.63093	29
30	56.08494	66.43885	79.05819	94.46079	113.28321	136.30754	164.49402	30
31	59.32834	70.76079	84.80168	102.07304	123.34587	149.57522	181.94342	31
32	62.70147	75.29883	90.88978	110.21815	134.21354	164.03699	201.13777	32
33	66.20953	80.06377	97.34316	118.93343	145.95062	179.80032	222.25154	33
34	69.85791	85.06697	104.18375	128.25876	158.62667	196.98234	245.47670	34
35	73.65222	90.32031	111.43478	138.23688	172.31680	215.71075	271.02437	35
36	77.59831	95.83632	119.12087	148.91346	187.10215	236.12472	299.12681	36
37	81.70225	101.62814	127.26812	160.33740	203.07032	258.37595	330.03949	37
38	85.97034	107.70955	135.90421	172.56102	220.31595	282.62978	364.04343	38
39	90.40915	114.09502	145.05846	185.64029	238.94122	309.06646	401.44778	39
40	95.02552	120.79977	154.76197	199.63511	259.05652	337.88245	442.59256	40
41	99.82654	127.83976	165.04768	214.60957	280.78104	369.29187	487.85181	41
42	104.81960	135.23175	175.95054	230.63224	304.24352	403.52813	537.63699	42
43	110.01238	142.99337	187.50758	247.77650	329.58301	440.84566	592.40069	43
44	115.41288	151.14301	199.75803	266.12085	356.94965	481.52177	652.64076	44
45	121.02939	159.70016	212.74351	285.74931	386.50562	525.85873	718.90484	45
46	126.87057	168.68516	226.50812	306.75176	418.42607	574.18602	791.79532	46
47	132.94539	178.11942	241.09861	329.22439	452.90015	626.86276	871.97485	47
48	139.26321	188.02539	256.56453	353.27009	490.13216	684.28041	960.17234	48
49	145.83373	198.42666	272.95840	378.99900	530.34274	746.86565	1057.18957	49
50	152.66708	209.34800	290.33590	406.52893	573.77016	815.08356	1163.90853	50
n	4%	5%	6%	7%	8%	9%	10%	n

Table 2 Present Value of an Annuity

$$PV = Ra_{\overline{n}|i}$$

n	$\frac{1}{2}\%$	1%	$1\frac{1}{2}\%$	2%	$2\frac{1}{2}\%$	3%	$3\frac{1}{2}\%$	n
1	0.99502	0.99010	0.98522	0.98039	0.97561	0.97087	0.96618	1
2	1.98510	1.97046	1.95588	1.94156	1.92742	1.91347	1.89969	2
3	2.97025	2.94099	2.91220	2.88388	2.85602	2.82861	2.80164	3
4	3.95050	3.90197	3.85438	3.80773	3.76197	3.71710	3.67308	4
5	4.92587	4.85343	4.78265	4.71346	4.64583	4.57971	4.51505	5
6	5.89638	5.79548	5.69719	5.60143	5.50813	5.41719	5.32855	6
7	6.86207	6.72819	6.59821	6.47199	6.34939	6.23028	6.11454	7
8	7.82296	7.65168	7.48593	7.32548	7.17014	7.01969	6.87396	8
9	8.77906	8.56602	8.36052	8.16224	7.97087	7.78611	7.60769	9
10	9.73041	9.47130	9.22218	8.98259	8.75206	8.53020	8.31661	10
11	10.67703	10.36763	10.07112	9.78685	9.51420	9.25266	9.00155	11
12	11.61893	11.25508	10.90751	10.57534	10.25776	9.95400	9.66333	12
13	12.55615	12.13374	11.73153	11.34837	10.98318	10.63496	10.30274	13
14	13.48871	13.00370	12.54338	12.10625	11.69091	11.29607	10.92052	14
15	14.41662	13.86505	13.34323	12.84926	12.38138	11.93794	11.51741	15
16	15.33993	14.71787	14.13126	13.57771	13.05500	12.56110	12.09412	16
17	16.25863	15.56225	14.90765	14.29187	13.71220	13.16612	12.65132	17
18	17.17277	16.39827	15.67256	14.99203	14.35336	13.75351	13.18968	18
19	18.08236	17.22601	16.42617	15.67846	14.97889	14.32380	13.70984	19
20	18.98742	18.04555	17.16864	16.35143	15.58916	14.87747	14.21240	20
21	19.88798	18.85698	17.90014	17.01121	16.18455	15.41502	14.69797	21
22	20.78406	19.66038	18.62082	17.65805	16.76541	15.93692	15.16712	22
23	21.67568	20.45582	19.33086	18.29220	17.33211	16.44361	15.62041	23
24	22.56287	21.24339	20.03041	18.91393	17.88499	16.93554	16.05837	24
25	23.44564	22.02316	20.71961	19.52346	18.42438	17.41315	16.48151	25
26	24.32402	22.79520	21.39863	20.12104	18.95061	17.87684	16.89035	26
27	25.19803	23.55961	22.06762	20.70690	19.46401	18.32703	17.28536	27
28	26.06769	24.31644	22.72672	21.28127	19.96489	18.76411	17.66702	28
29	26.93302	25.06579	23.37608	21.84438	20.45350	19.18845	18.03577	29
30	27.79405	25.80771	24.01584	22.39646	20.93023	19.60044	18.39205	30
31	28.65080	26.54229	24.64615	22.93770	21.39540	20.00043	18.73628	31
32	29.50328	27.26959	25.26714	23.46834	21.84918	20.38877	19.06887	32
33	30.35153	27.98969	25.87895	23.98856	22.29181	20.76579	19.39021	33
34	31.19555	28.70267	26.48173	24.49859	22.72379	21.13184	19.70068	34
35	32.03537	29.40858	27.07559	24.99862	23.14516	21.48722	20.00066	35
36	32.87102	30.10751	27.66068	25.48884	23.55625	21.83225	20.29049	36
37	33.70250	30.79951	28.23713	25.96945	23.95732	22.16724	20.57053	37
38	34.52985	31.48466	28.80505	26.44064	24.34860	22.49246	20.84109	38
39	35.35309	32.16303	29.36458	26.90259	24.73034	22.80822	21.10250	39
40	36.17223	32.83469	29.91585	27.35548	25.10278	23.11477	21.35507	40
41	36.98729	33.49969	30.45896	27.79949	25.46612	23.41240	21.59910	41
42	37.79800	34.15811	30.99405	28.23479	25.82061	23.70136	21.83488	42
43	38.60527	34.81001	31.52123	28.66156	26.16645	23.98190	22.06269	43
44	39.40823	35.45545	32.04062	29.07996	26.50385	24.25427	22.28279	44
45	40.20720	36.09451	32.55234	29.49016	26.83302	24.51871	22.49545	45
46	41.00219	36.72724	33.05649	29.89231	27.15417	24.77545	22.70092	46
47	41.79322	37.35370	33.55319	30.28658	27.46748	25.02471	22.89944	47
48	42.58032	37.97396	34.04255	30.67312	27.77315	25.26671	23.09124	48
49	43.36350	38.58808	34.52468	31.05208	28.07137	25.50166	23.27656	49
50	44.14279	39.19612	34.99969	31.42361	28.36231	25.72976	23.45562	50
n	$\frac{1}{2}\%$	1%	$1\frac{1}{2}\%$	2%	$2\frac{1}{2}\%$	3%	$3\frac{1}{2}\%$	n

n	4%	5%	6%	7%	8%	9%	10%	n
1	0.96154	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	1
2	1.88609	1.85941	1.83339	1.80802	1.78326	1.75911	1.73554	2
3	2.77509	2.72325	2.67301	2.62432	2.57710	2.53129	2.48685	3
4	3.62990	3.54595	3.46511	3.38721	3.31213	3.23972	3.16987	4
5	4.45182	4.32948	4.21236	4.10020	3.99271	3.88965	3.79079	5
6	5.24214	5.07569	4.91732	4.76654	4.62288	4.48592	4.35526	6
7	6.00205	5.78637	5.58238	5.38929	5.20637	5.03295	4.86842	7
8	6.73274	6.46321	6.20979	5.97130	5.74664	5.53482	5.33493	8
9	7.43533	7.10782	6.80169	6.51523	6.24689	5.99525	5.75902	9
10	8.11090	7.72173	7.36009	7.02358	6.71008	6.41766	6.14457	10
11	8.76048	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	11
12	9.38507	8.86325	8.38384	7.94269	7.53608	7.16073	6.81369	12
13	9.98565	9.39357	8.85268	8.35765	7.90378	7.48690	7.10336	13
14	10.56312	9.89864	9.29498	8.74547	8.24424	7.78615	7.36669	14
15	11.11839	10.37966	9.71225	9.10791	8.55948	8.06069	7.60608	15
16	11.65230	10.83777	10.10590	9.44665	8.85137	8.31256	7.82371	16
17	12.16567	11.27407	10.47726	9.76322	9.12164	8.54363	8.02155	17
18	12.65930	11.68959	10.82760	10.05909	9.37189	8.75563	8.20141	18
19	13.13394	12.08532	11.15812	10.33560	9.60360	8.95011	8.36492	19
20	13.59033	12.46221	11.46992	10.59401	9.81815	9.12855	8.51356	20
21	14.02916	12.82115	11.76408	10.83553	10.01680	9.29224	8.64869	21
22	14.45112	13.16300	12.04158	11.06124	10.20074	9.44243	8.77154	22
23	14.85684	13.48857	12.30338	11.27219	10.37106	9.58021	8.88322	23
24	15.24696	13.79864	12.55036	11.46933	10.52876	9.70661	8.98474	24
25	15.62208	14.09394	12.78336	11.65358	10.67478	9.82258	9.07704	25
26	15.98277	14.37519	13.00317	11.82578	10.80998	9.92897	9.16095	26
27	16.32959	14.64303	13.21053	11.98671	10.93516	10.02658	9.23722	27
28	16.66306	14.89813	13.40616	12.13711	11.05108	10.11613	9.30657	28
29	16.98371	15.14107	13.59072	12.27767	11.15841	10.19828	9.36961	29
30	17.29203	15.37245	13.76483	12.40904	11.25778	10.27365	9.42691	30
31	17.58849	15.59281	13.92909	12.53181	11.34980	10.34280	9.47901	31
32	17.87355	15.80268	14.08404	12.64656	11.43500	10.40624	9.52638	32
33	18.14765	16.00255	14.23023	12.75379	11.51389	10.46444	9.56943	33
34	18.41120	16.19290	14.36814	12.85401	11.58693	10.51784	9.60857	34
35	18.66461	16.37419	14.49825	12.94767	11.65457	10.56682	9.64416	35
36	18.90828	16.54685	14.62099	13.03521	11.71719	10.61176	9.67651	36
37	19.14258	16.71129	14.73678	13.11702	11.77518	10.65299	9.70592	37
38	19.36786	16.86789	14.84602	13.19347	11.82887	10.69082	9.73265	38
39	19.58448	17.01704	14.94907	13.26493	11.87858	10.72552	9.75696	39
40	19.79277	17.15909	15.04630	13.33171	11.92461	10.75736	9.77905	40
41	19.99305	17.29437	15.13802	13.39412	11.96723	10.78657	9.79914	41
42	20.18563	17.42321	15.22454	13.45245	12.00670	10.81337	9.81740	42
43	20.37079	17.54591	15.30617	13.50696	12.04324	10.83795	9.83400	43
44	20.54884	17.66277	15.38318	13.55791	12.07707	10.86051	9.84909	44
45	20.72004	17.77407	15.45583	13.60552	12.10840	10.88120	9.86281	45
46	20.88465	17.88007	15.52437	13.65002	12.13741	10.90018	9.87528	46
47	21.04294	17.98102	15.58903	13.69161	12.16427	10.91760	9.88662	47
48	21.19513	18.07716	15.65003	13.73047	12.18914	10.93358	9.89693	48
49	21.34147	18.16872	15.70757	13.76680	12.21216	10.94823	9.90630	49
50	21.48218	18.25593	15.76186	13.80075	12.23348	10.96168	9.91481	50
n	4%	5%	6%	7%	8%	9%	10%	n

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